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EXISTENCE OF SOLUTIONS OF SOME NONLINEAR WAVE EQUATIONS

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0. Introduction and Theorem

Let H be a real Hilbert space and A be a positive self adjoint operator in H. Let ϕ be a lower semi continuous proper convex function from H to $(-\infty,\infty]$ and $\partial\phi$ be the subdifferential of ϕ. Then we shall consider the following equation

$$(0.1)\qquad \begin{cases} \dfrac{d^2}{dt^2}u+Au+\partial\phi u\ni f(\cdot, u) \\ u(0)=a\,, \quad \dfrac{d}{dt}u(0)=b \qquad \text{on} \quad [0, T] \end{cases}$$

where T is a positive number.

The above equation was studied in Schatzman [3], [4], [5] and Maruo [2]. In this paper we prove the existence of a solution of the problem (0.1) under certain assumptions which are somewhat weaker than those of Schatzman [5] and Maruo [2].

In [5] Schatzman showed the existence and uniqueness of a solution of the following nonlinear wave equation

$$(0.2)\qquad \begin{cases} \left(\dfrac{\partial^2}{\partial t^2}u-\dfrac{\partial^2}{\partial x^2}u\right)(u-r)=0\,, \quad \dfrac{\partial^2}{\partial t^2}u-\dfrac{\partial^2}{\partial x^2}u\geqq 0 \\ \text{in the sense of distributions in} \quad [0, 1]\times[0, T], \\ u(x, t)\geqq r(x)\,, \quad u(x, 0)=u_0(x)\,, \quad \text{for} \quad x\in[0, 1]\,, \\ \dfrac{\partial}{\partial t}u(x, t)=u_1(x) \quad \text{a.e.} \quad \text{in } [0, 1]\,, \\ u(0, t)=u(1, t)=0 \qquad \text{for} \quad t\in[0, T]\,, \end{cases}$$

where r is a continuous given function such that $r(0)<0$, $r(1)<0$ and $\dfrac{d^2}{dx^2}r(x)\geqq 0$ (in the distribution sense). Set $K=\{f\in L_2(0, 1); f(x)\geqq r(x)\}$. The equation (0.2) is rewritten as the following equation in $L_2(0, 1)$