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VARIATION OF THE BERGMAN KERNEL BY CUTTING A HOLE

In memory of Professor Hitoshi Kumano-go

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Introduction

The present paper is concerned with the variation of the Bergman kernel of a bounded domain in $\boldsymbol{C}^n$ by cutting a *hole* that is, a relatively closed subset such that the remaining open set is connected and non-empty. In case a hole is small enough, we shall present an explicit formula representing the Bergman kernel of the holed domain in terms of that of the original domain, where the required smallness of the hole will be also determined exactly. A similar problem has been discussed in Schiffer-Spencer [32], §6.15.

Recently, much progress has been made on the study of the dependence of the Bergman kernel on a domain which is assumed to be strictly pseudoconvex, see Fefferman [17], [18] and the references therein; see also Greene-Krantz [20], [21] and Bergman-Schiffer [13], Komatsu [28] for the variation under a smooth perturbation of the boundary. Contrary to these works, our present method is elementary and results are of general character; in particular, no smoothness of the boundary will be required.

The Bergman kernel $K_\Omega(z, w)$ for $z, w\in\Omega$ of a bounded domain Ω in $\boldsymbol{C}^n$ is the reproducing kernel of the space $L^2H(\Omega)$ of square integrable holomorphic functions in Ω, so that it involves, in principle, all information on $L^2H(\Omega)$. Hence, if a hole ω of Ω is so small that the restriction mapping R: $L^2H(\Omega) \to L^2H(\Omega\backslash\omega)$ has a dense range, then one may expect that $K_{\Omega\backslash\omega}(z, w)$ is expressed in terms of $K_\Omega(\cdot, \cdot)$. This is indeed the case; in fact, we shall show that

$$K_{\Omega\backslash\omega}(z, w) = K_\Omega(z, w) + \sum_{m=1}^{\infty} T_{\Omega,\omega}^{(m)}(z, w) \qquad \text{for } z, w\in\Omega\backslash\omega\,,$$

where each $T_{\Omega,\omega}^{(m)}(z, w)$ for $z, w\in\Omega$ is given by

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