Iwasaki, C.
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CONSTRUCTION OF THE FUNDAMENTAL SOLUTION FOR DEGENERATE PARABOLIC SYSTEMS AND ITS APPLICATION TO CONSTRUCTION OF A PARAMETRIX OF $\square_b$

Dedicated to the memory of Professor Hitoshi Kumano-go

CHISATO IWASAKI

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Introduction. This paper intends to study the fundamental solution $E(t)$ of a degenerate parabolic system of pseudo-differential operators:

$$\text{(0.1)}\qquad \begin{cases} \left[\dfrac{d}{dt}+p(x, D)\right] E(t)=0\,, \quad t>0\,, \quad x\in \boldsymbol{R}^n\,, \\ E(0)=I\,, \end{cases}$$

where $k\times k$ matrix $p(x, \xi)$ has the following expansion:

$$\text{(0.2)}\qquad \begin{aligned} & p(x, \xi)=p_m(x, \xi)+p_{m-1}(x, \xi)+p_{m-2}(x, \xi)\,, \\ & p_{m-j}\in S^{m-j}_{1,0} \quad (j=0, 1, 2)\,, \\ & p_{m-j}(x, \lambda\xi)=\lambda^{m-j}\, p_{m-j}(x, \xi) \ \ \lambda>0,\ \xi\neq 0 \quad (j=0, 1) \end{aligned}$$

and $\qquad m>1$.

Our aim is to find $E(t)$ in some class of pseudo-differential operators. We adopt the Weyl symbol for pseudo-differential operators in this paper. The main theorem of this paper is that one can construct the fundamental solution $E(t)$ in the class $S^0_{1/2,1/2}$ of pseudo-differential operators with parameter t provided the symbol (0.2) satisfies the following Condition (A):

Condition (A).

$$\text{(A)–(i)}\quad p_m(x, \xi)=q_m(x, \xi)\, I\,,$$

where q_m $(\in S^m_{1,0})$ is a non-negative scalar symbol.

$$\text{(A)–(ii)}\quad \min_{1\leqslant j\leqslant k}(\mathrm{Re}\,\mu_j(x, \xi))+\tilde{\mathrm{tr}}\, A/2\geqslant c\,|\xi|^{m-1}$$