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ON THE EXISTENCE OF INTERSECTIONAL LOCAL TIME EXCEPT ON ZERO CAPACITY SET

Dedicated to the memory of Professor Takehiko Miyata

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0. Introduction

Let W be the space of $\boldsymbol{R}^d$-valued continuous functions on [0, 1], where $d\geqq 2$. We shall consider the functionals on W

$$\psi(\alpha, w) = \frac{d-\alpha}{4}\int_0^1\int_0^1 |w(t)-w(s)|^{-\alpha}\, dsdt\,, \qquad \alpha<2\,, \tag{0.1}$$

which may take infinite value. These functionals play an important role in the investigation of properties of function w: the finiteness of $\psi(\alpha, w)$ implies that the Hausdorff dimension of range $\{w(t);\ 0\leqq t\leqq 1\}$ is no less than α (cf. Taylor [9]). Let Q be the Wiener measure on W. Since $\psi(\alpha, w)$ is finite Q-almost surely for any $\alpha<2$, the Hausdorff dimension of $\{w(t);\ 0\leqq t\leqq 1\}$ is no less than 2 Q-almost surely.

Next, let α tend to 2. Though the mean of $\psi(\alpha, \cdot)$ with respect to Q diverges to infinity, the functional

$$\Psi_n(w) = \psi(2-2^{-n}, w)-2^n \tag{0.2}$$

converges Q-almost surely. In case $d=2$, Varadhan studied this limit functional in connection with the quantum field theory and proved its existence (cf. Appendix to Symanzik [8]).

Recently Fukushima [1] showed that various famous properties of sample paths such as Lévy's Hölder continuity hold not only Q-almost surely but also *quasi-everywhere, i.e.* except on a set of zero capacity with respect to the Ornstein–Uhlenbeck process on W. On the other hand, Kôno [4] and [5] proved that if $d\leqq 4$, then sample paths are recurrent with positive capacity. Therefore 'quasi-everywhere' is strictly finer than 'Q-almost everywhere'.

The purpose of this paper is to show that $\psi(\alpha, w)$ is finite quasi-everywhere for any $\alpha<2$ and that $\lim \Psi_n(w)=\Psi(w)$ exists quasi-everywhere. The former result implies the theorem in Komatsu and Takashima [3]: the Hausdorff