Kunita, H.
Osaka J. Math.
21 (1984), 883–911

ON THE CONVERGENCE OF SOLUTIONS OF STOCHASTIC ORDINARY DIFFERENTIAL EQUATIONS AS STOCHASTIC FLOWS OF DIFFEOMORPHISMS

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(Received November 28, 1983)

Introduction

Let $F(\tau, x, t, \omega)$ and $G(\tau, x, t, \omega)$ be random vector fields on R^d with time parameters τ, t satisfying certain statistical properties. Let ψ_t^ε be a solution of the stochastic ordinary differential equation

$$\frac{dx}{dt} = \varepsilon F(\varepsilon^2 t, x, t, \omega) + \varepsilon^2 G(\varepsilon^2 t, x, t, \omega).$$

A lot of attention has been shown to the limiting behavior of the solution ψ_t^ε as $\varepsilon \to 0$ and $t \to \infty$ with $\varepsilon^2 t$ remaining fixed, since the work of Khasminskii [8]. See also [3], [7], [13], [14], [16]. In these works, it is proved that $\phi_t^\varepsilon \equiv \psi_{t/\varepsilon^2}^\varepsilon$ converges weakly to a diffusion process ϕ_t with local characteristics a^{ij} and b^i which are determined from random vector fields $F(\tau, x, t, \omega)$ and $G(\tau, x, t, \omega)$ in a suitable way. (See (1.6)–(1.8) of Section 1). Note that ϕ_t^ε satisfies

$$\frac{d}{dt}\phi_t^\varepsilon = F_\varepsilon(t, \phi_t^\varepsilon),$$

where

$$F_\varepsilon(t, x) = \frac{1}{\varepsilon} F(t, x, \frac{t}{\varepsilon^2}) + G(t, x, \frac{t}{\varepsilon^2}).$$

The purpose of this paper is to show that the weak limit of ϕ_t^ε satisfies a suitable Itô's stochastic differential equation, which can be regarded as the weak limit of the above stochastic ordinary equation. Indeed, we will see in Theorem 1 that setting $X_t^\varepsilon(x) = \int_0^t F_\varepsilon(s, x)ds$, the pair $(\phi_t^\varepsilon, X_t^\varepsilon)$ converges weakly to (ϕ_t, X_t), where ϕ_t is a diffusion process mentioned above and X_t is a Brownian motion with values in the space of vector fields. Furthermore, these two processes are linked by Itô's stochastic differential equation