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ON THE BOUNDARY BEHAVIOR OF THE DIRICHLET SOLUTIONS AT AN IRREGULAR BOUNDARY POINT

Dedicated to Professor Makoto Ohtsuka on his 60th birthday

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Introduction. In the classical potential theory, O. Frostman [2] investigated the boundary behavior of the Dirichlet solution H_f^U for continuous boundary data f at an irregular boundary point x of a bounded domain U of R^n. And it was revealed that the cluster set of H_f^U at x is a segment with a possible exception. In other words, the cluster set of harmonic measures at x has two extreme points —the Dirac measure ε_x and the balayaged measure $\varepsilon_x^{\complement U}$. A generalization of this result was given by Constantinescu-Cornea [1] in an axiomatic setting in a more comprehensive context. Recently, J. Lukeš–J. Malý [6] considered this problem in a relatively compact open subset of a harmonic space. The present paper is a contribution to this problem under a resolutive compactification.

Let X be a $\mathcal{P}$-harmonic space with countable base in the sense of Constantinescu–Cornea [1] and X^* be a resolutive compactification. Let U be an open set of X. The closure $\bar{U}$ of U in X^* is a resolutive compactification of U. Suppose that $\partial U=(\bar{U}\setminus U)\cap X\neq\emptyset$. For a sequence $\{b_k\}$ converging to $x\in\partial U$ and satisfying $\varepsilon_{b_k}^{\complement U}\to\varepsilon_x^{\complement U}$, the harmonic measure of U at b_k converges to a measure λ_x. If x is irergular for $\bar{U}$, λ_x enjoyes remarkable properties stated in Theorem 7, which has a counterpart with the results of Lukeš-Malý [6] and Hyvönen [3], and is connected with a version of maximal sequences considered by Smyrnélis [7]. In view of the work of Lukeš-Malý, we can decide the structure of the cluster set $\mathcal{M}_x^U$ of harmonic measures and reveal that the type of $\mathcal{M}_x^U$ is a local property. We can also conclude the same result for the cluster set of the normalized Dirichlet solutions.

1. Preliminaries

Let X be a $\mathcal{P}$-harmonic space with countable base in the sense of Constantinescu-Cornea [1] and X^* be a resolutive compactification of X. We assume that there exists a function s_0 which is bounded superharmonic on X and $\inf s_0>0$. We write $\Delta=X^*\setminus X$.