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A NOTE ON REDUCED FORMS OF EFFECTIVELY HYPERBOLIC OPERATORS AND ENERGY INTEGRALS

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1. Introduction

Let P be the principal symbol of a hyperbolic differential operator. At a double characteristic point of P, its Taylor expansion begins with the quadratic form in the cotangent bundle. The coefficient matrix of the Hamiltonian system associated with this quadratic form is called the fundamental (or Hamilton) matrix. If the fundamental matrix has non-zero real eigenvalues, P is said to be effectively hyperbolic operator ([1], [2]).

Ivrii and Petkov conjectured in [2] that C^∞ Cauchy problem for effectively hyperbolic operators is well posed for any lower order term; that is effectively hyperbolic operator is strongly hyperbolic.

In this note, in §2, we reduce effectively hyperbolic operators of second order to certain standard forms by homogeneous canonical transformations. Since we are concerned with the Cauchy problem, we shall use only homogeneous canonical transformations which do not depend on the time and its dual variables. In §3, for some simple but essential examples, we indicate how the standard forms relate to the energy integrals which assure the strong hyperbolicity.

The detailed proofs of deriving the energy estimates for effectively hyperbolic operators of the standard forms will be appear elsewhere.

Denote $x^{(p)}=(x_p, \cdots, x_d)$, $\xi^{(p)}=(\xi_p, \cdots, \xi_d)$, $x=x^{(0)}$, $\xi=\xi^{(0)}$, $0\leq p\leq d$, and consider

$$P(x, \xi) = \xi_0^2-Q(x, \xi^{(1)}),$$

where $Q(x, \xi^{(1)})$ is defined in a conic neighborhood of $(0, \bar{\xi}^{(1)})$, non-negative and homogeneous of degree 2 in $\xi^{(1)}$.

Let $(0, \bar{\xi})$ be a double characteristic point of $P(x, \xi)$. That is $dP(x, \xi)$ vanishes at $(0, \bar{\xi})$. This is the same thing as $\bar{\xi}=(0, \bar{\xi}^{(1)})$, $Q(0, \bar{\xi}^{(1)})=0$. Denote by $F_P(x, \xi)$ the fundamental matrix evaluated at (x, ξ) (for the precise definition, see [2]). In the following, $\{,\}$ denotes the Poisson bracket. The