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ON FREE BOUNDARY PLATEAU PROBLEM FOR GENERAL DIMENTIONAL SURFACES

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The aim of this paper is to deal with a free boundary Plateau problem following Reifenberg's method.

The so-called Plateau problem of seeking surfaces of least area bounded by a prescribed contour has been attacked from various aspects. Classical approaches due to Radó, Douglas and Courant have limited the problem to the case of dimension two, where the admissible surface is a mapping image of some fixed parameter domain. Reifenberg is one of the first who considered minimal surfaces, with a given boundary, of various topological types simultaneously. In his pioneering paper [11], [12] and [13] he fixed a compact subset A of R^p and regarded any compact set X containing A as a surface with boundary A so far as X spans A homologically.

In the present paper we are concerned with a free boundary problem which is not discussed in Reifenberg's studies. It will be natural to define the free boundary of a surface homologically. We formulate a free boundary problem approximately and prove existence and regularity results with some simple examples illustrating our situation. The result of this paper remains valid in case the ambient space is a Riemannian manifold, which is close to the Euclidean space in the sense of Morrey [10], especially, a compact Riemannian manifold.

1. Formulation of free boundary problem

In this section we formulate the free boundary problem mentioned above. Suppose that we are given a compact subset E of R^p. It is just the set on which all our free boundaries should lie. For any compact subset X of R^p, we define $FB(X):=X\cap E$ to be free boundary of X on E. Then the inclusion maps

$$(1) \qquad i\colon FB(X)\hookrightarrow X\,, \quad j\colon FB(X)\hookrightarrow E$$

induce the following diagram