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EQUIVARIANT POINT THEOREMS FOR FIBRE-PRESERVING MAPS

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1 Introduction

Let $p: X\to B$ and $p': X'\to B'$ be local trivial fibre spaces with fibre-preserving involutions $T: X\to X$ and $T': X'\to X'$ respectively, and let $f: X\to X'$ be a fibre-preserving map. Denote by A_f the set of equivariant points of f:

$$A_f = \{x\in X; fT(x) = T'f(x)\}\ ,$$

and by $\bar{A}_f$ its orbit space under T. In this paper we shall study $H^*(\bar{A}_f)$ in connection with $H^*(B)$, where H^* is the Čech cohomology with coefficients in Z_2. Two theorems will be proved by making use of the technique of establishing a transfer homomorphism, which was initiated by Becker and Gottlieb ([1], [2])

In case $p: X\to B$ is an m-sphere bundle with the antipodal involution and $p': X'\to B$ is an R^n-bundle with the trivial involution, Jaworowski gave in [4], [5] the following theorem which is a "continuous" version of the Borsuk-Ulam theorem: If $k=m-n\geqq 0$ and the all the Stiefel-Whitney classes of $p': X'\to B$ are zero then the composition

$$H^i(B)\xrightarrow{\bar{p}^*} H^i(\bar{A}_f)\xrightarrow{\smile\omega(A_f)^k} H^{i+k}(\bar{A}_f)$$

is injective for every i, where $\bar{p}: \bar{A}_f\to B$ is induced by $p|A_f$, and $\omega(A_f)$ is the characteristic class of the double covering $A_f\to\bar{A}_f$. It is seen in this paper that the assumption on the Stiefel-Whitney classes is superfluous in the theorem of Jaworowski.

Throughout this paper we use the Čech cohomology with coefficients in Z_2.

2 Equivariant fundamental cohomology class

Let $M\to X\xrightarrow{p} B$ be a local trivial fibre space such that both the fibre M and the base B are manifolds without boundary. Suppose that there is given a fibre-preserving involution $T: X\to X$, that is, an involution satisfying $pT=T$.