Minami, H.
Osaka J. Math.
21 (1984), 789-808

ON THE K-THEORY OF SO(n)

Dedicated to Professor Nobuo Shimada on his 60th birthday

HARUO MINAMI

(Received November 9, 1983)

The purpose of this paper is to compute $KO^*(SO(n))$ for $n\equiv -1, 0, 1$ mod 8 and $K^*(SO(n))$ as algebras, where $SO(n)$ is the rotation group of degree n.

The ring $K^*(SO(n))$ has been already determined in [6], [8], [10] and [15]. The calculations in these papers are based on the theorem of Hodgkin in [9] for the K-group of the spinor group Spin(n). Here using also this theorem and the Thom isomorphism theorem we shall show that there exists a short exact sequence in the equivariant K-theory associated with Z_2 involving the injection $K^*(SO(n))\to K^*(P^{n-1})\otimes K^*(\mathrm{Spin}(n))$ as in [8] where P^l is the real projective l-space, and making use of this exact sequence we shall give a proof of the result on $K^*(SO(n))$.

Using the theorem of Hodgkin, Seymour has proved in [14] a theorem on the additive structure of $KO^*(\mathrm{Spin}(n))$. From his result we have a similar short exact sequence in the equivariant KO-theory as in the complex case. So we shall next determine the algebra structure of $KO^*(SO(n))$ for n as above by arguments parallel to $K^*(SO(n))$. Then we shall use the result of Crabb in [4] on the squares of elements in $KO^{-1}(X)$.

Throughout this paper an A-module generated by x is denoted by $A\cdot x$ where A is a ring.

1. Preliminaries

a) By G we denote the multiplicative group $\{-1, 1\}$ throughout this paper. Let $R^{p,q}$ be the euclidean space R^{p+q} with a G-action such that -1 reverses the first p coordinates and fixes the last q. Let $S^{p,q}$ and $B^{p,q}$ be the unit sphere and unit ball in $R^{p,q}$ and $\Sigma^{p,q}=B^{p,q}/S^{p,q}$ with the collapsed $S^{p,q}$ as base point. Thus -1 acts on $S^{k,0}$ as antipodal involution and $P^{k-1}=S^{k,0}/G$. We consider that G acts on Spin(m) as the subgroup $\{-1, 1\}$ of Spin(m) and $\mathrm{Spin}(m)/G=SO(m)$ (see [11]). We denote the natural projections $S^{k,0}\to P^{k-1}$ and $\mathrm{Spin}(m)\to SO(m)$ by the same letter π.

For any $x=(x_1, \cdots, x_k)$ of R^k we write $x=x_1e_1+ \cdots +x_ke_k$ as a vector where