Ōshima, H.
Osaka J. Math.
21 (1984), 765–772

A REMARK ON JAMES NUMBERS OF STIEFEL MANIFOLDS

Dedicated to Professor Nobuo Shimada on his 60th birthday

HIDEAKI ŌSHIMA

(Received July 13, 1983)

1. Introduction

The purpose of this note is to supply a few relations between the unstable and stable James numbers of Stiefel manifolds.

Let F be the field H of the quaternions or the field C of the complex numbers, and d the dimension of F over the field of the real numbers. Let $G(F^n)$ be the symplectic group $Sp(n)$ or the unitary group $U(n)$ according as F is H or C. The stunted quasi-projective space $Q_{n,k}=Q_n/Q_{n-k}$ is a subspace of the Stiefel manifold $O_{n,k}=G(F^n)/G(F^{n-k})$ (see e.g. [8]). There exist the quotient maps $q_r\colon Q_{n,k}\to Q_{n,k-r}$ and $p_r\colon O_{n,k}\to O_{n,k-r}$. Let $i'\colon Q_{n,k}\to O_{n,k}$ be the inclusion map. Then $i'\circ q_r=p_r\circ i'$ and $i'\colon Q_{n,1}\to O_{n,1}$ is the identity map of the $(dn-1)$-dimensional sphere S^{dn-1}.

Applying the homotopy functor $\pi_{dn-1}(\ )$ and the stable homotopy functor $\pi^s_{dn-1}(\ )$ to q_{k-1} and p_{k-1}, we define the unstable James numbers (see [7]) $Q\{n,k\}=Q_F\{n,k\}$, $O\{n,k\}=O_F\{n,k\}$ and the stable James numbers $Q^s\{n,k\}=Q^s_F\{n,k\}$, $O^s\{n,k\}=O^s_F\{n,k\}$ by the following equations:

$$q_{k-1*}\pi_{dn-1}(Q_{n,k})=Q\{n,k\}\pi_{dn-1}(S^{dn-1}),$$
$$p_{k-1*}\pi_{dn-1}(O_{n,k})=O\{n,k\}\pi_{dn-1}(S^{dn-1}),$$
$$q_{k-1*}\pi^s_{dn-1}(Q_{n,k})=Q^s\{n,k\}\pi^s_{dn-1}(S^{dn-1}),$$
$$p_{k-1*}\pi^s_{dn-1}(O_{n,k})=O^s\{n,k\}\pi^s_{dn-1}(S^{dn-1});$$

whenever $1\leqq k\leqq n$. As easily seen (see e.g. [12]), we have

(1.1) $\quad Q^s\{n,k\}\mid Q\{n,k\}$, $O^s\{n,k\}\mid O\{n,k\}$, $O\{n,k\}\mid Q\{n,k\}$, $Q^s\{n,k\}\mid Q^s\{n,k+1\}$, $Q\{n,k\}\mid Q\{n,k+1\}$ *and* $O\{n,k\}\mid O\{n,k+1\}$;

where $a\mid b$ means that b is a multiple of a. In [12] we proved

$$Q^s\{n,k\}=O^s\{n,k\}. \tag{1.2}$$

The stable James number $O^s\{n,k\}$ has been investigated by various au-