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PSEUDO-RIEMANNIAN SYMMETRIC R-SPACES

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Introduction. Symmetric R-spaces are specific riemannian symmetric spaces. These spaces are canonically realized as complete connected full parallel submanifolds of euclidean spaces and as minimal submanifolds of certain hyperspheres in the euclidean spaces. Conversely, a complete connected full parallel submanifold is congruent to the product image of imbeddings homothetic to the canonical imbeddings of symmetric R-spaces (Ferus [3], Takeuchi [18]). Roughly speaking, symmetric R-spaces are constructed as follows. Take a non-degenerate Jordan triple system which is compact and let $(\mathfrak{g}, \rho)$ be the positive definite symmetric graded Lie algebra constructed from its Jordan triple system in the Koecher's fashion, i.e., $\mathfrak{g}=\mathfrak{g}_{-1}\oplus\mathfrak{g}_0\oplus\mathfrak{g}_1$ is a graded Lie algebra of non compact type and ρ is a Cartan involution of $\mathfrak{g}$. Let $\mathfrak{g}=\mathfrak{k}\oplus\mathfrak{p}$ be the Cartan decomposition. Then there exists a unique element $\nu\in\mathfrak{p}$ such that $\mathfrak{g}_\mu$, $\mu=0, \pm 1$, are eigen spaces of $\mathrm{ad}(\nu)$ with eigen values ν respectively. Then a symmetric R-space is defined as the orbit of ν by the connected Lie subgroup of $GL(\mathfrak{g})$ with the Lie algebra $\mathrm{ad}(\mathfrak{k})|\mathfrak{p}$. Here the euclidean metric on $\mathfrak{p}$ is given by the restriction of the Killing form of $\mathfrak{g}$ to $\mathfrak{p}$.

Naitoh [11] has defined notions "orthogonal Jordan triple system", "orthogonal symmetric graded Lie algebra". An orthogonal Jordan triple system $(V, \{\ \}, \langle\ \rangle)$ is a Jordan triple system $(V, \{\ \})$ with a non-degenerate symmetric bilinear form $\langle\ \rangle$ on V. Non-degenerate Jordan triple systems are orthogonal Jordan triple systems with their trace forms. An orthogonal symmetric graded Lie algebra $(\mathfrak{g}, \rho, \langle\ \rangle_{\mathfrak{p}})$ is a symmetric graded Lie algebra $(\mathfrak{g}, \rho)$ with a non-degenerate symmetric bilinear form $\langle\ \rangle_{\mathfrak{p}}$ on $\mathfrak{p}$. Semi-simple symmetric graded Lie algebras are orthogonal symmetric graded Lie algebras with the restrictions of the Killing forms of $\mathfrak{g}$ to $\mathfrak{p}$. Between these objects there exists a natural one-to-one correspondence, which is the extension of the Koecher's way. (See § 1 for these precise definitions and the correspondence.) Moreover, in the above paper, we have constructed pseudo-riemannian

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