Mashimo, K.
Osaka J. Math.
21 (1984), 721-732

MINIMAL IMMERSIONS OF 3-DIMENSIONAL SPHERE INTO SPHERES

KATSUYA MASHIMO

(Received April 8, 1983)
(Revised March 19, 1984)

Introduction

Let S^n_c be the n-dimensional sphere with constant curvature c. Let Δ be the Laplace-Beltrami operator on S^n_1. The spectre and eigen-functions of Δ are well-known [2]. Let V^d be the eigen-space of Δ corresponding to the d-th eigen-value $\lambda_d=d(d+n-1)$. Let $f_0, f_1, \cdots, f_{m(d)}$ be an orthonormal basis of V^d with respect to the inner product. Then

$$\begin{aligned}\psi_{n,d}\colon\ & S^n_{k(d)} \to S^{m(d)}_1(\subset \boldsymbol{R}^{m(d)+1}) \\ & ;\ p \to 1/(m(d)+1)(f_0(p), f_1(p), \cdots, f_{m(d)}(p))\,,\end{aligned}$$

is an isometric minimal immersion, where $k(d)$ and $m(d)$ are as follows [6];

$$\begin{aligned}k(d) &= n/d(d+n-1)\,, \\ m(d) &= (2d+n-1)(d+n-2)!/d!(n-1)!-1\,.\end{aligned}$$

It is proved that any isometric minimal immersion of S^2_c into S^N_1 is equivalent to $\psi_{2,d}$ for some d, [3], [6]. But it is not true if the dimension n is greater than 3. In fact do Carmo and Wallach proved the following

Theorem 0.1 (do Carmo and Wallach, [7]). *Let $f\colon S^n_c \to S^N_1$ be an isometric minimal immersion. Then*
(i) *there exists an integer d such that $c=k(d)$.*
(ii) *There exists a positive semi-definite matrix A of size $(m(d)+1)\times(m(d)+1)$ such that f is equivalent to $A\circ\psi_{n,d}$.*
(iii) *If $n=2$ or $d\leqq 3$, then A is the identity matrix.*
(iv) *If $n\geqq 3$ and $d\geqq 4$, then A is parametrized by a compact convex body L in some finite dimensional vector space,* $\dim L\geqq 18$. *If A is an interior point of L then $N=m(d)$, and if A is a boundary point of L then $N<m(d)$.*

There are some problems concerning (iv) of the above Theorem.