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ON THE STRUCTURE OF THE AUGMENTATION QUOTIENTS RELATIVE TO AN N_p-SERIES

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1. Introduction

Let G be a group with lower central series $G=G_1\supseteqq G_2\supseteqq G_3\supseteqq\cdots\supseteqq G_n\supseteqq G_{n+1}\supseteqq\cdots$, and define

$$W_n(G)=\sum\bigotimes_{i=1}^{n} Sp^{a_i}(G_i/G_{i+1}),$$

where $\sum$ runs over all non-negative integers $a_1, a_2, \cdots, a_n$ such that $\sum ia_i=n$, and $Sp^{a_i}(G_i/G_{i+1})$ is the a_i-th symmetric power of the abelian group G_i/G_{i+1}. Let $I(G)$ be the augmentation ideal of G in $\boldsymbol{Z}G$. We denote by $Q_n(G)$ the additive groups $I^n(G)/I^{n+1}(G)$ for $n\geqq 1$. Some results are known about the structure of $Q_n(G)$.

It is well known that $Q_1(G)\simeq W_1(G)$ for any group G. G. Losey [3] proved that $Q_2(G)\simeq W_2(G)$ for any finitely generated group G. Tahara [6], [7] proved that $Q_3(G)\simeq W_3(G)/R_4^*$ and $Q_4(G)\simeq W_4(G)/R_5^*$ hold for any finite group G, where R_4^* and R_5^* are precisely determined subgroups of $W_3(G)$ and $W_4(G)$. Furthermore Sandling and Tahara [5] proved that $Q_n(G)\simeq W_n(G)$ $(n\geqq 1)$ if G_i/G_{i+1} is free abelian for any $i\geqq 1$.

Let p be a prime number. In the first half of this paper we restrict our attention to groups of exponent p, and prove that

$$Q_n(G)\simeq W_n(G)/R_{n+1} \qquad (n\geqq 1),$$

where R_{n+1} is a precisely determined subgroup of $W_n(G)$ (Theorem 8). As its corollaries we have a well known result 1), and a new result 2) as follows:

1) $D_n(G)=G_n$ for any such group G, where $D_n(G)$ is the n-th dimension subgroup of G (Corollary 9).

2) Let G be a finite group with lower central series

$$G=G_1\supseteqq G_2\supseteqq\cdots\supseteqq G_c\supseteqq G_{c+1}=1.$$

If this series is an N_p-series then $Q_n(G)\simeq W_n(G)$ for $n<p$ (Remark 12).

In the latter half we prove that $Q_p(G)\simeq W_p(G)$ if the lower central series of G is an N_p-series (Theorem 13). Furthermore we construct a subgroup