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ON A CONSTRUCTION OF EXCEPTIONAL PSEUDOSYMMETRIC SETS

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1. Introduction

If we denote the conjugation $b^{-1}ab$ by $a\circ b$ or $a^{\sigma(b)}$ in a group, it satisfies (i) $\sigma(b)$ is a permutation on the group, (ii) $a\circ a=a$, and (iii) $(a\circ b)\circ c=(a\circ c)\circ(b\circ c)$. Under the condition (i), (iii) is equivalent with a fundamental identity: $\sigma[b^{\sigma(c)}]=\sigma(c)^{-1}\sigma(b)\sigma(c)$. The present author called a binary system satisfying these conditions a pseudosymmetric set in [2] and [3]. D. Joyce called it a quandle in [1]. When every $\sigma(b)$ is of order 2, a pseudosymmetric set is called a symmetric set (or an involutive quandle in [1], or a symmetric groupoid in [4].) A pseudosymmetric set is called *special* if it is isomorphic with a pseudosymmetric subset of a group (=a pseudosymmetric set in the above sense). It is easily seen that a special pseudosymmetric set is isomorphic with a union of conjugacy classes of some group. Its structure is determined to some extent by its enveloping group ([3]). A pseudosymmetric set which is not special is called *exceptional* by analogy with the thoery of Jordan algebras. The simplest exceptional pseudosymmetric set is a set $\{a, b, c\}$ with $\sigma(a)=\sigma(b)=1$ and $\sigma(c)\neq 1$ ([4], p. 72). The structure of general pseudosymmetric sets seems to be more complicate, and our consideration will be restricted to the transitive one; a pseudosymmetric set is called transitive when the group of permutations generated by all $\sigma(b)$ is a transitive permutation group on the set. The object of this paper is to show a construction method of transitive exceptional pseudosymmetric sets. In this direction, S. Doro discussed about the existence of transitive exceptional symmetric sets in his unpublished paper. (See the remark in the last part of this paper.) For the construction, we need an extension theory of pseudosymmetric sets, which will be given in **2**. Let S be a pseudosymmetric set and A a set (a trivial pseudosymmetric set). We intend to make $A\times S$ a pseudosymmetric set by introducing a suitable composition. For (a, s), $(b, t)\in A\times S$, we consider a composition $(a, s)\circ(b, t)=(c, s\circ t)$, where generally c depends on a, b, s and t, satisfying the three conditions of a pseudosymmetric set. But in this paper we condiser a simple but important case that c depends only on a, s and t. Denote $c=a^{\pi(s,t)}$. $\pi(s, t)$ is a permutation