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ON THE NONEXISTENCE OF UNKNOWN PERFECT 6- AND 8-CODES IN HAMMING SCHEMES $H(n, q)$ WITH q ARBITRARY

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0. Introduction

The first important study for the (non-)existence of perfect e-codes in the Hamming schemes $H(n, q)$ with arbitrary q was made by E. Bannai [1]. In his paper, Bannai determined the asymptotic locations of the zeros of Lloyd's polynomials as $\beta=\sqrt{(n-e)(q-1)}/q\rightarrow\infty$. (See 1.3.). Derived from this, he proved that, for each e, there exists a number $\beta_0(e)$ such that if $\beta\geqslant\beta_0(e)$, then there is no nontrivial perfect e-code in $H(n, q)$ for $q>2$. In this paper, we will use Bannai's idea and explictly calculate such numbers $\beta_0(e)$ for $e=6$ and 8. Namely, we will prove that we can take $\beta_0(6)=15$ and $\beta_0(8)=18$ under the assumption that $q\geqslant 30$. The remaining cases $\beta<\beta_0(e)$ (and $q\geqslant 30$) are also treated. Since the cases $q<30$ are already determined (see 1.2.), we then get the following theorem.

Theorem A. *There exists no nontrivial perfect e-code in Hamming schemes $H(n, q)$ for $e=6$ or 8 with q arbitrary.*

As explained in section 1.2., the nonexistence of nontrivial perfect e-codes in $H(n, q)$ for all $e\geqslant 3$ was almost completed by Best [2]. He used Bannai's idea [1] to prove this nonexistence for $e=7$ and $e\geqslant 9$. The cases $e=3$, 4, and 5 were previously solved by Reuvers [7]. Thus theorem A fills the gap (of $e=6$ and 8) and we get:

Theorem B (see 1.1.2 and 1.2). *For $e\geqslant 3$, the only perfect e-codes in $H(n, q)$ are the trivial codes* (of size 1 or 2) *and the binary Golay code* ($q=2$, $n=23$, $e=3$).

We conclude this section with the following open problem.

For $e=1$ or 2, the existence or classification of perfect e-codes still remains open. As far as the author knows, for $e=2$, only the ternary Golay code ($q=3$, $n=11$, $e=2$) is known. For $e=1$, there are many of them known [12], and the classification seems very difficult.