Kado, J.
Osaka J. Math.
21 (1984), 683-686

THE FIXED SUBRINGS OF A FINITE GROUP OF AUTOMORPHISMS OF $\aleph_0$-CONTINUOUS REGULAR RINGS

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(Received June 13, 1983)

Let R be an associative ring, G a finite group of automorphisms of R, and let R^G be the fixed subring of G on R. A. Page has proved that if R is a left self-injective regular ring and the order $|G|$ of G is invertible in R, then R^G is also a left self-injective regular ring [8]. This theorem is very useful when we investigate some structure of a nonsingular ring and the fixed subring of a finite group of automorphisms.

Recently D. Handelman has discovered an $\aleph_0$-continuous regular ring which coordinates the lattice of projections of a finite Rickart C^*-algebra as a subring of the maximal quotient ring of its C^*-algebra [4]. We shall prove in this note the following generalization of Page's theorem: if R is a left $\aleph_0$-continuous, left $\aleph_0$-injective regular ring and $|G|$ is invertible in R, then R^G is again a left $\aleph_0$-continuous, $\aleph_0$-injective regular ring. We shall show as a corollary that if R is a left $\aleph_0$-continuous regular ring with $|G|^{-1}\in R$, R^G is a left $\aleph_0$-continuous regular ring and S^G is the maximal left $\aleph_0$-quotient ring of R^G, where S is the maximal left $\aleph_0$-quotient ring of R.

1. Skew group rings

DEFINITION [7]. Let R be a ring with identity element 1 and G a finite group of automorphisms of R. The skew group ring, $R*G$, is defined to be a free left R-module with basis $\{g: g\in G\}$ and multiplication given as follows: if $r, s\in R$ and $g, h\in G$, then $(rg)(sh)=rs^{g^{-1}}gh$.

DEFINITION [3]. A regular ring R is left $\aleph_0$-continuous if the lattice of principal left ideals of R is upper $\aleph_0$-continuous. A ring T is left $\aleph_0$-injective if every homomorphism from a countably generated left ideal of T into T is extendable to a T-module endomorphism of T. For modules A and B, $A\subset_e B$ implies that A is an essential submodule of B.

A regular ring R has a maximal left $\aleph_0$-quotient ring S which is a quotient ring defined by the filter-like set $\mathfrak{X}$ consisting of all countably generated, essen-