Kobayashi, S.
Osaka J. Math.
21 (1984), 679–682

A NOTE ON REGULAR SELF-INJECTIVE RINGS

SHIGERU KOBAYASHI

(Received March 25, 1983)
(Revised June 17, 1983)

Introduction

Let R be an arbitrary ring and let M be a finitely generated right R-module. Then the dual module $M^*=\mathrm{Hom}_R(M, R)$ of M is a left R-module. If R is a left noetherian ring or finite dimensional algebra, then M^* is also finitely generated. But this property does not hold for a general ring. However, we prove that R has this property if R is a regular right self-injective ring (Proposition).

The purpose of this paper is to prove the following theorems.

Theorem 1. *Let R be a regular ring. Then the following statements are equivalent.*
1) *R is a right and left self-injective ring.*
2) *For every finitely generated non-singular right (resp. left) R-module, the dual module is a non-zero finitely generated left (resp. right) R-module.*

In particular, if R is a commutative regular ring, then we have the following theorem.

Theorem 2. *Let R be a commutative regular ring. Then the following conditions are equivalent.*
1) *R is a self-injective ring.*
2) *For every finitely generated R-module, the dual module is also finitely generated.*

Throughout this paper, we assume that R is a ring with identity element and all modules are unitary. We denote the maximal right quotient ring of R by Q.

Let M be a right R-module. Then we denote the right (resp. left) annihilator ideal by $r(M)$ (resp. $l(M)$), i.e. $r(M)=\{r\in R \mid Mr=0\}$, (resp. $l(M)=\{r\in R \mid rM=0\}$).

We denote the category of right R-modules by Mod-R. Let M be a right R-module. Then M is said to be a *cogenerator* in Mod-R if $\mathrm{Hom}_R(-, R)$ is a faithful functor. In particular, if R is an injective cogenerator in Mod-R, then R is said to be a *right PF*-ring.