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ON MAXIMAL SUBMODULES OF A FINITE DIRECT SUM OF HOLLOW MODULES II

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Introduction

We have given, in [3], the structure of right artinian rings satisfying the following conditions: i) the Jacobson radical of a ring is square zero and ii) every submodule of a direct sum of hollow (local) modules is also a direct sum of hollow modules. The latter property cited above implies that every maximal submodule of a direct sum of $t+1$-copies of a hollow module with length t contains a direct summand.

In this paper, we shall study this property for any right artinian ring, and reproduce, in §1, the results similar to ones in [3] without the assumption that the Jacobson radical is square zero. In §2 we shall give a characterization of some rings in terms of the property above.

1 Property (**)

Let R be a ring with identity. In this paper, every R-module is a unitary right R-module. Let M be an R-module. We shall denote the Jacobson radical of M by $J(M)$ and the radical of R by J or $J(R)$, respectively. Throughout this paper we assume that R is a right artinian (semi-perfect) ring and every R-module M has the finite composition length, which we denote by $|M|$. If M has a unique maximal submodule $J(M)$, M is called *hollow* (*local*). In this case $M\approx eR/A$ for a primitive idempotent e and a right ideal A in eR.

Given a family $\boldsymbol{N}=\{N_i\}_{i=1}^t$ of (hollow) modules, we denote by $D(\boldsymbol{N})$ the direct sum $\sum_{i=1}^t \oplus N_i$. If $N_i=N$ for a fixed module N, we indicate this by $N^{(t)}$. We have studied in [3] the following property:

(**) *Every maximal submodule of $D(\boldsymbol{N})$ contains a non-zero direct summand of $D(\boldsymbol{N})$.*

Since the above property is preserved by Morita equivalence, we may assume that R is a basic ring. Hence, from now on, we assume that R is a right artinian and basic ring. Let N be a hollow module with finite length. We put $\bar{N}=N/J(N)$, and S $(=S_N)=\mathrm{End}_R(N)$. Then $\Delta=\mathrm{End}_R(\bar{N})$ is a division