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ON MAXIMAL SUBMODULES OF A FINITE DIRECT SUM OF HOLLOW MODULES I

Dedicated to Professor Hirosi Nagao on his 60th birthday

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Introduction

We have given, in [5], a characterization of a right (upper) serial ring in terms of submodules of finite direct sums of serial modules. In this paper we shall replace the serial module by hollow modules in the above. Then it is clear that we shall be able to obtain a new class of rings R generalized from the right serial rings.

However, it is difficult for the author to give a complete characterization of those rings. We shall restrict ourselves to a particular case where the Jacobson radical J of R is square zero. It is not still easy to find the characterization of such rings. If R is either a commutative artinian ring or an algebra of finite dimension over an algebraically closed field, then we can show the structure of R as follows: $|eR|$, the composition length of eR, is equal to or less than three and if two simple right ideals A_1 and A_2 in eJ are isomorphic to each other, then there exists a unit element x in eRe such that $A_2=xA_1$ and $[eRe/eJe: \Delta(A_1)]_r=2$, where e is a primitive idempotent of R and $\Delta(A_1)=\{\bar{x}\in eRe/eJe \mid \bar{x}A_1\subseteq A_1\}$. We shall give the similar structure for any right artinian ring R under an assumption that $|eR|\leqslant 5$. We do not know any examples of rings which have the property mentioned above (see Condition I in § 3) and $|eR|\geqslant 4$ for some primitive idempotent e. We shall study the similar problem without the assumption $eJ^2=0$ in the forthcomming paper.

1 Right serial rings

Let R be a ring with identity. Every module in this paper is a unitary right R-module. For an R-module M, $|M|$ means the length of the composition series of M. We shall denote the *Jacobson radical* and the *socle* of M by $J(M)$ and $S(M)$, respectively. Put $J^n(M)=J(J^{n-1}(M))$ and $S_n(M)/S_{n-1}(M) =S(M/S_{n-1}(M))$ inductively. Then $M\supseteq J(M)\supseteq J^2(M)\supseteq\cdots$ and $0\subseteq S_1(M)\subseteq S_2(M)\subseteq\cdots$ are called the *upper Loewy series* and the *lower Loewy series* of M, respectively. If each factor module $J^n(M)/J^{n+1}(M)$ $(S_{n+1}(M)/S_n(M))$ is simple