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TACHIKAWA'S THEOREM ON ALGEBRAS OF LEFT COLOCAL TYPE

Dedicated to Professor Hirosi Nagao on his 60th birthday

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Introduction

Let A be an artinian ring. Then A is said to be of right local type if any finitely generated indecomposable right A-module M is local (i.e. M has a unique maximal submodule) and a ring of left colocal type is defined as the dual notion. We say A is left serial if a left A-module A is a direct sum of uniserial submodules. Tachikawa [4, 5] gave characterizations of algebras of right local (or equivalently of left colocal) type.

Theorem (Tachikawa). *For a finite dimensional algebra A with the Jacobson radical N, the following conditions* (a)–(d) *are equivalent.*

(a) *A is of right local type.*

(b) *A is of left colocal type.*

(c) (c_1) *A is left serial.*

(c_2) *For any uniserial left A-modules L_1 and L_2 with $|L_1|\leq|L_2|$, any isomorphism θ: $S_1(L_1)\to S_1(L_2)$ is (L_1, L_2)-maximal or (L_1, L_2)-extendible* (*see Section* 1 *for the definitions*), *where $|L_i|$ is the composition length of L_i and $S_1(L_i)$ is the socle of L_i for $i=1, 2$.*

(c_3) $|eN/eN^2|\leq 2$ *for any primitive idempotent e of A.*

(d) (d_1) *A is left serial.*

(d_2) $eN=M_1\oplus M_2$ *for any primitive idempotent e of A, where M_i is either zero or a uniserial submodule of the right A-module eN for each $i=1, 2$.*

More precisely Tachikawa [4] gave a proof of the equivalence of (b) and (c) for *any* artinian ring. But in the proof of the implication from (c) to (b), there were two gaps. He himself pointed out one of them, namely [4, Lemma 4.9], and informed Fuller of it and that the lemma holds for any artinian ring under a suitable assumption (D) which is satisfied for any finite dimensional algebra over a field (cf. Section 3 for the definition of (D). See also Fuller [3, Note p. 165].). Now the other one (which is related to [4, Corollary 4, 6])