Shigekawa, I.
Osaka J. Math.
21 (1984), 621-627

ON A QUASI EVERYWHERE EXISTENCE OF THE LOCAL TIME OF THE 1-DIMENSIONAL BROWNIAN MOTION

ICHIRO SHIGEKAWA*)

(Received November 5, 1983)

1. Introduction

Recently quasi everywhere properties of the Brownian motion were discussed by many authors; Williams considered the quadratic variation (see [9]) and Fukushima [3] considered the nowhere differentiability, Lévy's Hölder continuity, the law of iterated logarithm etc. By the way, the *local time* plays an important role in stochastic analysis. The existence of the local time of the 1-dimensional Brownian motion was proved by Trotter [10]. He proved that the local time of the 1-dimensional Brownian motion exists almost everywhere (a.e.) with respect to the Wiener measure. In this paper we shall prove that it exists *quasi everywhere* (q.e.) with respect to the *Ornstein-Uhlenbeck process* on the Wiener space.

Fukushima's study is based on a concept of *capacity* related to the Ornstein-Uhlenbeck process. The term "quasi everywhere" means "except on a set of capacity 0". A set of capacity 0 is characterized by the Ornstein-Uhlenbeck process as follows (see [2], [6]). Let W_0^1 be a set of all continuous paths w: $[0, \infty)\rightarrow \boldsymbol{R}$ vanishing at 0 with the compact uniform topology and μ be the Wiener measure on W_0^1. Let $(X_\tau)_{\tau\geqq 0}$ be a W_0^1-valued Ornstein-Uhlenbeck process with the initial distribution μ defined on an auxiliary probability space $(\Omega, \mathcal{F}, P)$. Then for any $A\subset W_0^1$, A is of capacity 0 if and only if

$$(1.1) \qquad P[X_\tau\notin A \quad \text{for all } \tau>0] = 1\,.$$

On the other hand, by the Tanaka formula the local time $(\phi(\tau, t, a))$ of a Brownian motion $(X_\tau(t))_{t\geqq 0}$ is given by

$$\phi(\tau, t, a) = (X_\tau(t)-a)^+-(X_\tau(0)-a)^+-\int_0^t 1_{(a,\infty)}(X_\tau(s))X_\tau(ds)$$

(cf. [4], [8]). Then our main theorem is stated as follows.

*) This research was partially supported by Grant-in-Aid for Scientific Research.