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THE HAUSDORFF DIMENSION OF QUASI-ALL BROWNIAN PATHS

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1. Introduction. Properties of Brownian paths have been main subjects of many papers and various results have been already well known. The size of sample path, for example, has been investigated by evaluating Lebesgue measure and Hausdorff measure etc. of path. Taylor [7] determined the Hausdorff dimension of almost all Brownian paths. On the other hand, Fukushima [2] recently studied some basic properties of path, such as nowhere-differentiability, Lévy's Hölder continuity, the law of iterated logarithm etc. in connection with the Dirichlet space theory on the Wiener space. He proved that these properties hold not only almost surely but also *quasi-everywhere*.

In this paper we shall present a refinement of Taylor's result from the viewpoint of the Dirichlet space theory. It is easily derived from a combination of the definition of Hausdorff measure and Fukushima's result corresponding to Lévy's Hölder continuity that the Hausdorff dimension of path is no more than 2 quasi-everywhere. We shall prove that the Hausdorff dimension is no less than 2 quasi-everywhere, by showing that a specific Wiener functional used in [7] belongs not only to L^2-space but also to the Dirichlet space.

2. Dirichlet space. We first prepare some notations and definitions on the Wiener space and a Dirichlet space on it (cf. Shigekawa [6] and Fukushima [2]).

Let W be the Banach space of all $\boldsymbol{R}^d$-valued continuous functions $w(t)$ on $[0, 1]$ satisfying $w(0)=0$, with standard supremum norm, H be the Hilbert space of all absolutely continuous functions of W having square integrable derivatives, with inner product

$$\langle h, g\rangle_H = \int_0^1 \dot{h}(t)\cdot\dot{g}(t)\,dt\,,$$

and norm $\|h\|_H=\sqrt{\langle h, h\rangle_H}$, where $\dot{h}$ denotes the derivative of h and $a\cdot b$ denotes the inner product in $\boldsymbol{R}^d$. Let P be the Wiener measure on W. A Wiener functional is a real (or more generally Hilbert space) valued mapping defined on W, measurable with respect to the Borel field of W. Let $L^2(W)$ be the