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ON THE PERTURBATION THEORY FOR FREDHOLM OPERATORS

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Introduction

In the classical theory of linear Fredholm operators a fundamental role is played by compact operators. In fact, a condition for an operator to be Fredholm is given in terms of compact operators, and it is known that the class of Fredholm operators is invariant under compact perturbations. The object of this paper is to generalize these results introducing a new class of operators containing the class of compact operators.

The organization of the paper is as follows: in the section 1 we introduce some basic definitions, propositions and examples; in the section 2 we establish the aimed results and in the section 3 we present an application of Theorem 2.2.

1. Let X and Y be Banach spaces.

DEFINITION 1.1. An operator $T: D(T)\subset X\rightarrow X$ is said to be demicompact if for every bounded sequence $\{x_n\}$ in $D(T)$ such that $x_n-Tx_n\rightarrow x_0$, for some x_0 in X, as $n\rightarrow\infty$, then there is a convergent subsequence of $\{x_n\}$.

Here $D(T)$ denotes the domain of T.

Examples of demicompact operators.

a) Compact operators $T: D(T)\subset X\rightarrow X$ are demicompact.
If X is a Hilbert space,

b) Operators $T: D(T)\subset X\rightarrow X$ which satisfy either the condition

$$\mathrm{Re}(Tx-Ty,\ x-y)\leq a\|x-y\|^2,\ a<1 \tag{1}$$

or the condition

$$\mathrm{Re}(Tx-Ty,\ x-y)\leq a\|Tx-Ty\|^2,\ a<1 \tag{2}$$

are demicompact.

c) Operators $T: D(T)\subset X\rightarrow X$ for which $(I-T)^{-1}$ exists and is continuous on its range $R(I-T)$ (and, in particular, demicontinuous operators T for which

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