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ON THE RATE OF CONVERGENCE FOR MAXIMUM LIKELIHOOD ESTIMATES IN A TRUNCATED CASE

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1. Introduction. Let $X_1, \cdots, X_n$ be independent random variables with common density $f(x-\theta)$, $-\infty<x, \theta<\infty$, where θ is an unknown translation parameter. We shall consider here the case that $f(x)$ is a uniformly continuous density which vanishes on the interval $(-\infty, 0]$ and is positive on the interval $(0, \infty)$ and particularly

$$f(x)\sim\alpha\beta x^{\alpha-1} \text{ as } x\to+0 \tag{1.1}$$

with $1<\alpha<2$ and $\beta>0$.

Let $\hat{\theta}_n=\hat{\theta}_n(X_1, \cdots, X_n)$ denote the MLE (maximum likelihood estimate) of θ for the sample size n. Woodroofe [7] showed that $(\beta n)^{1/\alpha}(\hat{\theta}_n-\theta)$ has a limiting distribution which is not the normal distribution. Furthermore, he studied asymptotic properties of this limiting distribution. In this paper, we shall investigate the rate of convergence to the limiting distribution for MLE. This result is applied to estimate the probability of moderate deviations for the distribution of MLE.

The case that (1.1) is satisfied with $\alpha\geqq 2$ has already studied by the author ([2], [3]). In [2], the author showed that if $\alpha>3$, $\sqrt{nB}\ (\hat{\theta}_n-\theta)$ converges uniformly to the standard normal distribution with the convergence order $O(n^{-1/2})$, and that if $2<\alpha\leqq 3$, the order of convergence to normality is $o(n^{-\nu/2})$ for every $\nu<(\alpha-2)/2$. Here B denotes Fisher's information number. In the case $\alpha=2$, it was shown in [3] that for every real number t there exists $C>0$ such that for all θ and $n\geqq 1$

$$|P_\theta\{\sqrt{\beta n(\log n+\log\log n)}\ (\hat{\theta}_n-\theta)\leqq t\}-\Phi(t)|\leqq C(\log n)^{-1},$$

where Φ denotes the standard normal distribution. It is noticed that the rate of convergence, which is uniform in t, is a little slower than the order $(\log n)^{-1}$.

In the regular case, it is well known that the same result as in the case of (1.1) with $\alpha>3$ holds (see Pfanzagl [5]). It is interesting to note that Takeuchi [6] has studied Edgeworth type expansion of the distribution of the sum of independent random variables in some non-regular cases.