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SOME REMARKS ON THE CAUCHY PROBLEM FOR SCHRÖDINGER TYPE EQUATIONS

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Dedicated to the memory of Professor Hitoshi Kumano-go

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0. Introduction

In the present paper we consider the Cauchy problem for the following equation

$$(0.1) \qquad Lu \equiv (i\partial_t + \tau\Delta + \sum_{j=1}^{m} b_j(x)\partial_{x_j} + c(x))u(x, t) = 0$$

with initial data $u_0(x)$ at $t=0$, where τ is a constant such that $0 \leqq \tau \leqq 1$, and $b_j(x)$, $c(x)$ belong to $\mathcal{B}^\infty(R^m_x)$. $\mathcal{B}^\infty(R^m_x)$ denotes the set of C^∞-functions whose derivatives of any order are all bounded. If τ is positive, the above equation (0.1) is the typical equation of non-kowalewskian type which is not parabolic. The study of the equation (0.1) is important for the study of equations of general non-kowalewskian type.

For real s let H_s be the Sobolev space with the usual norm $||\cdot||_s$ and let $H_\infty \equiv \bigcap_{s\in R} H_s$ be the Fréchet space with semi-norms $||\cdot||_s$, $s=0, \pm 1, \pm 2, \cdots$. We say that the Cauchy problem for (0.1) is well posed for the future (resp. for the past) in the space H_∞, if there exists a constant $T>0$ (resp. $T<0$) such that for any initial data $u_0(x) \in H_\infty$ a unique solution $u(x, t) \in \mathcal{E}^0_t([0, T]; H_\infty)$ of (0.1), which takes $u_0(x)$ at $t=0$, exists. Here, $f(x, t) \in \mathcal{E}^0_t([0, T]; H_\infty)$ means that the mapping: $[0, T] \ni t \to f(x, t) \in H_\infty$ is continuous in the topology of H_∞.

Our purpose is to prove the following theorem corresponding to the so-called Lax-Mizohata theorem for equations of kowalewskian type (Lax [5], Mizohata [6]).

Theorem. *In order that equation* (0.1) *is well posed for the future or for the past in the space* H_∞, *it is necessary that there exist constants* M *and* N *such that the inequality*

$$(0.2) \qquad \sup_{x\in R^m, \omega\in S^{m-1}} \left| \sum_{j=1}^{m} \int_0^\rho \mathrm{Re}\, b_j(x+2\tau\theta\omega)\omega_j d\theta \right| \leqq M \log(1+\rho) + N$$

holds for any $\rho \geqq 0$. S^{m-1} *denotes the unit sphere in* R^m.