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PARALLEL PROJECTIVE MANIFOLDS AND SYMMETRIC BOUNDED DOMAINS

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Introduction. The parallel complex submanifolds, i.e., complex subfolds with parallel second fundamental form, of Fubini-Study spaces were classified by Nakagawa-Takagi [12] and Takeuchi [20]. The first classification [12] was done as an application of the general study of Kähler immersions of locally symmetric Kähler manifolds into Fubini-Study spaces. The second one [20] was done by the determination (Takagi-Takeuchi [18]) of degrees of Kähler immersions of symmetric Kähler manifolds into Fubini-Study spaces.

In this paper we give another way of classification of such submanifolds. Let D be an irreducible symmetric bounded domain and V the holomorphic tangent space of D at a point $p\in D$. Then the isotropy group K at p acts in a natural way on the complex projective space $P(V)$ associated to V. We endow $P(V)$ with a K-invariant Kähler metric with positive constant holomorphic sectional curvature. Take a highest weight vector v of the irreducible K-module V. Then

$$M = K\cdot[v]\subset P(V),$$

where $[v]$ denotes the line $\boldsymbol{C}v$, is a complete full complex submanifold with parallel second fundamental form. This is proved by writing the second fundamental form of M in terms of the Lie algebra of infinitesimal automorphisms of D.

Conversely, any complete full complex submanifold M of a Fubini-Study space $P_N(\boldsymbol{C})$ with parallel second fundamental form is obtained in this way. This is proved by defining a structure of Jordan triple system on $\boldsymbol{C}^{N+1}$ making use of second fundamental form and curvature tensor of M, and then using Koecher's classification theorem for symmetric bounded domains by Jordan triple systems.

As an application, we study the group $\mathrm{Aut}(S)$ of automorphisms of a non-singular hyperplane section S of M. We show that $\mathrm{Aut}(S)$ is reductive if and only if the symmetric bounded domain D corresponding to M is a unit ball or of tube type. This provides a unified construction of compact complex manifolds admitting no Einstein Kähler metric found by Hano [2], Sakane