Matumoto, T.
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ON A WEAKLY UNKNOTTED 2-SPHERE IN A SIMPLY-CONNECTED 4-MANIFOLD

TAKAO MATUMOTO

Dedicated to Professor Minoru Nakaoka on his sixtieth birthday

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Introduction

The purpose of this note is to present the following criterion for unknotting in a weak sense which gives us a simple geometric proof of Theorem of Kawauchi stated below.

Theorem 1. *Let M be a smooth 1-connected 4-manifold and S^2 a smoothly embedded 2-sphere in M. Suppose that $\pi_1(M-S^2)\cong Z$ and $S^2\simeq 0$ in M. Then, S^2 is unknotted in $M\#(\underset{n}{\#}S^2\times S^2)$ for some $n\geqq 0$.*

Here S^2 is called unknotted if there is a smoothly embedded D^3 which is bounded by S^2. As a corollary we shall give a proof of Theorem of Kawauchi. His original proof uses the partial Poincaré duality associated to infinite cyclic covering (see [3], [4] and Suzuki [9, Th. 8.6]). Other proofs are founded in [1], [8] and [10].

Corollary (Theorem of Kawauchi). *Let S^2 be a smoothly embedded 2-sphere in the 4-sphere S^4. Suppose that $\pi_1(S^4-S^2)\cong Z$. Then, it is algebraically unknotted, i.e. $S^4-S^2\simeq S^1$.*

Is a smooth 2-knot with $\pi_1(S^4-S^2)\cong Z$ unknotted? This is a unsolved question. We stabilize the problem by making connected sum of the ambient manifold with $\#(S^2\times S^2)$ and another stabilization may be done by making connected sum of the embedded manifold S^2 with trivially embedded $\#(S^1\times S^1)$. There is a result due to [2].

Theorem 2 (Hosokawa-Kawauchi [2]). *Under the same assumption of Theorem 1, S^2 surgered by attaching n trivially embedded 1-handles is unknotted in M for some $n\geqq 0$.*

We refer the reader to [ibid] for the precise meaning of trivial (=trivially embedded) 1-handles and unknottedness of surfaces. We shall give also a