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A REMARK ON ODD-PRIMARY COMPONENTS OF SPECIAL UNITARY GROUPS

Dedicated to Professor Minoru Nakaoka on his 60th birthday

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Let G be a compact connected Lie group of dimension $d>0$, and let us assume that an orientation of G is chosen. Let $\mathfrak{L}$ denote the left invariant framing of the tangent bundle of G. For the pair $(G, \mathfrak{L})$ we obtain by the Pontrjagin-Thom construction an element $[G, \mathfrak{L}]$ in π_d^S. Ossa[6] proved that $72[G, \mathfrak{L}]=0$. Of course this implies that the p-primary component of $[G, \mathfrak{L}]$ is zero for any prime $p>3$. As for information on the 3-primary part of general nature we have the following results of Becker-Schultz: For $G=SO(2n)$, $Spin(2n)$ or $U(n)$ the 3-primary component of $[G, \mathfrak{L}]$ is zero [2]. For the exceptional Lie groups Knapp[4] proved that the 3-primary component of $[F_4, \mathfrak{L}]$ is zero. In this note we give the following additional information:

*) *For* $n\equiv 0$ *or* 3 mod 4 *the* 3*-primary component of* $[SU(n), \mathfrak{L}]$ *vanishes.*

Let τ be an involutive automorphism of G and let K denote the closed subgroup of G consisting of all elements fixed by τ. Then using the equivariant stable homotopy theory for involutions, we have

Proposition 1. *If K is of odd codimension in G, then*

$$[G, \mathfrak{L}]_{(odd)} = 0$$

where $a_{(odd)}$ denotes the odd-primary part of a.

The assertion *) is an immediate corollary of this proposition. According to the classification theorem of irreducible Riemannian symmetric spaces, examples of Lie groups to which this proposition applies are $SU(4n)$, $SU(4n+3)$, $Spin(2n)$ and $SO(2n)$. For $SU(n)$ (resp. $Spin(n+1)$ and $SO(n+1)$) we adopt the involutive automorphism corresponding to the symmetric space of AI-type (resp. of BDII-type), whose fixed point set is of codimension $(n-1)(n+2)/2$ (resp. n).

In the final section we make a remark on the real Adams e-invariant e'_R and we show