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STRUCTURES OF THE HAKEN MANIFOLDS WITH HEEGAARD SPLITTINGS OF GENUS TWO

Dedicated to Professor Minoru Nakaoka on his sixtieth birthday

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1. Introduction

In this paper we will give a complete list of the closed, orientable 3-manifolds with Heegaard splittings of genus two and admitting non-trivial torus decompositions. We use the following notations.

D(n) (A(n), Mö(n) resp.): *the collection of the Seifert fibered manifolds the orbit manifold of each of which is a disk (annulus, Möbius band resp.) with* n *exceptional fibers.*

M_K (M_L resp.) : *the collection of the exteriors of the two bridge knots (links resp.).*

L_K : *the collection of the exteriors of the one bridge knots in lens spaces each of which admits a complete hyperbolic structure or admits a Seifert fibration whose regular fiber is not a meridian loop.*

For the definitions of the one bridge knots in lens spaces see section 5. Then our main result is

Theorem. *Let M be a closed, connected Haken manifold with a Heegaard splitting of genus two. If M has a nontrivial torus decomposition then either*

(i) *M is obtained from $M_1 \in D(2)$ and $M_2 \in L_K$ by identifying their boundaries where the regular fiber of M_1 is identified with the meridian loop of M_2,*

(ii) *M is obtained from $M_1 \in Mö(n)$ (n=0, 1 or 2) and $M_2 \in M_K$ by identifying their boundaries where the regular fiber of M_1 is identified with the meridian loop of M_2,*

(iii) *M is obtained from $M_1 \in D(n)$ (n=2 or 3) and $M_2 \in M_K$ by identifying their boundaries where the regular fiber of M_1 is identified with the meridian loop of M_2,*

(iv) *M is obtained from M_1, $M_2 \in D(2)$ and $M_3 \in M_L$ by identifying their boundaries where the regular fiber of M_i (i=1, 2) is identified with the meridian loop of M_2 or*

(v) *M is obtained from $M_1 \in A(n)$ (n=0, 1 or 2) and $M_2 \in M_L$ by ident-*