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LOCALIZATION OF BP-MODULE SPECTRA WITH RESPECT TO BP-RELATED HOMOLOGIES

Dedicated to Professor Minoru Nakaoka on his sixtieth birthday

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1. Introduction

BP is the Brown-Peterson spectrum for a fixed prime p. It is an associative and commutative ring spectrum whose homotopy is $BP_*=Z_{(p)}[v_1,\cdots,v_n,\cdots]$. Following Ravenel [9] we denote by L_n the localization with respect to $v_n^{-1}BP_*$-homology and by L_∞ that with respect to $\bigoplus_n v_n^{-1}BP_*$-homology. Then there is a tower

$$X\to L_\infty X\cdots\to L_nX\to L_{n-1}X\to\cdots\to L_0X$$

for each *CW*-spectrum X. A *CW*-spectrum X is said to be *harmonic* if $X=L_\infty X$, and *s-harmonic* if $X=\hat{L}_\infty X$ where we put $\hat{L}_\infty X=\varprojlim_n L_nX$. X is harmonic whenever it is s-harmonic. In this paper we study some properties of s-harmonic spectra. Especially we discuss $\hat{L}_\infty E$ when E is an associative *BP*-module spectrum which satisfies one or two of the following conditions:

I) *E_* is v_m-torsion for any $m<n$,*
II) *E_* is v_m-torsion for any $m>n$,*
III) *$BP_*/I_m\otimes_{BP_*}E_*$ is v_m-torsion free for any $m\leqq n$,*
IV) *$\mathrm{Tor}_m^{BP_*}(BP_*/I_m,E_*)$ is v_m-divisible for any $m<n$, and*
V) *$\mathrm{hom\,dim}_{BP_*}E_*\leqq n$.*

As such associative *BP*-module spectra we have $P(n)$, $k(n)$, $BP\langle n\rangle$, N_nBP and so on.

We show that an associative *BP*-module spectrum E is s-harmonic if $\mathrm{hom\,dim}_{BP_*}E_*$ is finite (Theorem 4.8). This implies Ravenel's result ([9, Theorem 4.4] or [6, Theorem 1.3]) that a p-local connective *CW*-spectrum X is harmonic if $\mathrm{hom\,dim}_{BP_*}BP_*X$ is finite (Corollary 4.9). However the finiteness assumption is not necessarily essential because $L_\infty BP\langle n\rangle$ is s-harmonic although $\mathrm{hom\,dim}_{BP_*}L_\infty BP\langle n\rangle_*$ is infinite for $n\geqq 1$ (Proposition 4.12).

We intend to describe elementary properties of s-harmonic spectra corresponding to those of harmonic spectra. The product of harmonic spectra is