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KÄHLER-EINSTEIN METRIC ON AN OPEN ALGEBRAIC MANIFOLD

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0. Introduction

In [10], S.-T. Yau proved that if M is a compact complex manifold with negative first Chern class, then there is a unique Kähler-Einstein metric with negative Ricci curvature up to a constant multiple. The condition "with negative first Chern class" is, by definition, to assume that there is a negative definite real closed (1,1)-form in the de Rham cohomology class of the first Chern class $c_1(M)$. By the fact that for a holomorphic line bundle E on a compact complex manifold M, any real closed (1,1)-form on M belonging to the first Chern class $c_1(E)$ is the curvature form of a Hermitian metric for E multiplied by $1/2\pi$ (See [6], pp. 148–150.), it is equivalent to assuming the existence of a volume form on M with negative definite Ricci form. Therefore, it is natural to suspect that in the non-compact version of Yau's theorem, the condition "with negative first Chern class" should be replaced by the existence of a volume form with negative Ricci form ω with some additional conditions to control the behavior of ω at infinity: for example, $-\omega$ defines a complete Kähler metric with bounded curvature on noncompact manifold under consideration. (In this paper, a Kähler metric is identified with its Kähler form.) In fact, in [2], S.-Y. Cheng and Yau proved that if Ω is a smooth bounded strongly pseudoconvex domain in C^n, then there is a complete Kähler-Einstein metric with negative Ricci curvature, which is invariant under biholomorphisms of Ω; the strong pseudoconvexity of Ω implies the existence of a volume form with the above properties. In this case, a model of such manifolds is the unit ball B^n in C^n with Poincaré-Bergman metric:

$$\sqrt{-1}\partial\bar{\partial}\log(1-|z|^2)\,.$$

The purpose of this paper is to prove the existence of a complete Kähler-Einstein metric with negative Ricci curvature on the complement of hypersurfaces of projective algebraic manifolds. In fact, we prove the following theorem.

Theorem 1. *Let $\bar{M}$ be a complex projective algebraic manifold and D an*