Murai, M.
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A NOTE ON THE NUMBER OF IRREDUCIBLE CHARACTERS IN A p-BLOCK OF A FINITE GROUP

Dedicated to Professor Hirosi Nagao on his 60th birthday

MASAFUMI MURAI

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Introduction

Let G be a finite group and n a divisor of $|G|$, the order of G. We set $L_n(G)=\{x\in G\,|\,x^n=1\}$. G. Frobenius gave the following conjecture.

(F): If $|L_n(G)|=n$, then $L_n(G)$ is a normal subgroup of G.

Concerning this problem the following results are known.

(1) In order to prove (F) we may assume that n and $|G|/n$ are relatively prime. (See [7], [12])

(2) Let G be a minimal counterexample to (F). Then G is a simple group. (See [1], [12], [17])

Since the classification of finite simple groups has been completed, it may be possible to verify (F) by checking simple groups, by using (2). Indeed such verifications have been carried out for certain classes of simple groups by several authors ([1], [8], [9], [14], [16]).

However, it may also be desirable to investigate (F) from a more general standpoint. The purpose of this paper is to provide an example of such investigations.

For a group G and a prime p, we denote by $G_{p'}$, the set of p'-elements, or p-regular elements, of G and by $|G|_p$ the highest power of p dividing $|G|$. We set $|G|_{p'}=|G|/|G|_p$. We are interested in the following special case of (F).

(F_p): *If $|G_{p'}|=|G|_{p'}$, then G is p-nilpotent.*

In this case Brauer and Nesbitt [3] found the following fact. Let c_{11} be the Cartan invariant corresponding to the principal representation (mod p) of G. Then if c_{11} is not larger than $|G|_p$, then (F_p) is true for G. So it may be interesting to know whether c_{11} is not larger than $|G|_p$ for a finite group. But Landrock [11] showed that c_{11} can be larger than $|G|_p$ for a certain group (see also [6], p. 168). In this paper we consider another generalization of (F_p). Let