Asashiba, H.
Osaka J. Math.
21 (1984), 343-364

ON ALGEBRAS OF SECOND LOCAL TYPE, II

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(Received March 7, 1983)

This is a continuation of the previous paper with the same title which will be referred to as [I]. Throughout the paper, A denotes a (left and right) artinian ring with identity 1, J its Jacobson radical and unless otherwise stated, all modules are (unital and) finitely generated.

Let n be any natural number. Then we say that A is of *right n-th local* (resp. *colocal*) *type* in case for every indecomposable right A-module M, the n-th top $\text{top}^n M:=M/MJ^n$ (resp. the n-th socle $\text{soc}^n M:=$ the left annihilator of J^n in M) of M is indecomposable.

In this paper, we first examine an artinian ring which is of both left and right n-th local type (in this case the artinian ring is said to be of *two-sided n-th local type* or simply *n-th local type*) and give some necessary and sufficient conditions to be of this type, in particular for an algebra, we characterize this type by a structure of A (2.5). Note that this type of rings include the class of serial rings ([4]). Next, we come back to the case $n=2$ and restrict our interest to the case where A is an algebra over an algebraically closed field k, and give some further necessary conditions for A to be of right 2nd local type (3.4). (It is shown in [I, Example 2] that the necessary conditions stated in [I, Theorem 1] are not sufficient for A to be of right 2nd local type.) These conditions contain the list of all possible "shapes" of indecomposable projective *right* A-modules. (That of indecomposable projective *left* A-modules follows directly from [I, Theorem 1].) As an application, we give some necessary and sufficient conditions for a left serial algebra over an algebraically closed field to be of right 2nd local type (4.1). It should be noted that by [I, Theorem 1], an algebra over an algebraically closed field which is of right 2nd local type is left serial if every indecomposable projective left A-module P is of height≥ 4 (i.e. $J^3P\neq 0$). We remark that these theorems remain valid also in the case where the base field k is a splitting field for A. The last section is devoted to some examples.

The author would like to thank Professor T. Sumioka for his useful advice and Professor M. Harada for his careful reading of the preprint.