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ON ALGEBRAS OF SECOND LOCAL TYPE, I

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Throughout this paper, A denotes a (left and right) artinian ring with identity 1, J its Jacobson radical and all modules are (unital and) finitely generated.

Let n be any natural number. Then we say that A is of *right n-th local type* in case for every indecomposable right A-module M, the n-th top $\text{top}^n M:=M/MJ^n$ of M is indecomposable. (Note that if $\text{top}^n M$ is indecomposable, then so is M since A is artinian and M is finitely generated.) Hence for such a ring A, the question of indecomposability of right A-modules can be reduced to the corresponding problem of right A/J^n-modules. In [11] H. Tachikawa has studied the case $n=1$ and obtained a necessary and sufficient condition for algebras (by algebra we always mean a finite dimensional algebra over a field k) to be of this type. Further the representation theory of algebras with square-zero radical is well known [5], [6], [7]. So in this paper, we examine the case $n=2$ and give some necessary conditions for rings with selfduality to be of this type. Further in particular for QF (=quasi-Frobenius) rings, we give necessary and sufficient conditions to be of this type. More precisely, we show the following two theorems:

Theorem 1. *Let A be a ring with selfduality which is of right 2nd local type and e any primitive idempotent in A. Then*

(1) *J^2e is a uniserial waist in Ae if $J^2e\neq 0$* (see section 2 for definition of a waist),

(2) *eJ^m is a direct sum of local modules for every $m\geqslant 2$,*

(3) *for each local direct summand L of eJ^2, LJ^2 is uniserial (thus eJ^4 is a direct sum of uniserial modules).*

Further if A is an algebra, we have

(4) *Ae is uniserial if $h(Ae)\geqslant 5$.*

In particular if the base field k is, in addition, an algebraically closed field, then

(5) *Ae is uniserial if $h(Ae)\geqslant 4$,*

and then

(6) *eJ^2 is a direct sum of uniserial modules.*