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A TRANSFORMATION OF A SYMMETRIC MARKOV PROCESS AND THE DONSKER-VARADHAN THEORY

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1. Introduction

In obtaining the lower bound in the celebrated law of large deviation of the occupation distribution for the one dimensional Brownian motion, Donsker and Varadhan [3] performed a transformation of the absorbing Brownian motion on an interval (a, b) by a drift $b=(\log \rho)'$, or equivalently, by a multiplicative functional

$$\frac{\rho(B_t)}{\rho(B_0)}\exp\left(-\int_0^t \frac{\rho''}{2\rho}(B_s)ds\right)I_{\{t<\zeta\}}$$

into a conservative process on (a, b) with invariant probability measure $\rho^2 dx$, to which the ergodic theorem was well applied. Here ρ^2 is assumed to be a probability density C^2-function on R^1, positive inside (a, b) and vanishing outside.

We show in this paper that their method works for any symmetric Hunt process corresponding to a regular and irreducible Dirichlet form. In the present general case, we take function ρ from the range of the resolvent. In order to prove the conservativeness of transformed process, we make full use of an explicit expression of the transformed Dirichlet form, while the Feller test of non-explosion was available in the special case of [3].

We consider a locally compact separable metric space X and a positive Radon measure m on X such that $\mathrm{Supp}[m]=X$. The inner product in real L^2 space $L^2(X; m)$ is denoted by $(\ ,\ )$ and $C_0(X)$ stands for the space of continuous functions on X with compact support. Let $\boldsymbol{M}=(X_t, \boldsymbol{P}_x, \zeta)$ be a Hunt process on X which is *m-symmetric* in the sense that the transition function P_t satisfies $(P_t f, g)=(f, P_t g)$, $f, g\in C_0(X)$. Then the *Dirichlet form* E of M can be defined by $F=D[E]=D(\sqrt{-A})$, $E(u, v)=(\sqrt{-A}u, \sqrt{-A}v)$ where A is the infinitesmal generator of the semigroup on $L^2(X; m)$ determined by P_t. We always assume that E is regular: $F\cap C_0(X)$ is dense both in F and in $C_0(X)$,

In § 2, we derive the Beurling-Deny formula