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ASYMPTOTIC SUFFICIENCY II: TRUNCATED CASES

TADAYUKI MATSUDA

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1. Introduction. Asymptotic sufficiency of maximum likelihood (m.l.) estimator in regular cases has been studied by many authors (see Wald [17], LeCam [2], Pfanzagl [12], Michel [8], Suzuki [14], [15], and so on).

In [6], Matsuda showed that for $k\in N=\{1, 2, \cdots\}$ a statistic $T_{n,k}=(T_n, G_n^{(2)}(z_n, T_n), \cdots, G_n^{(k)}(z_n, T_n))$ is asymptotically sufficient up to order $O(n^{-k/2})$. Here $\{T_n\}$ is a sequence of asymptotic m.l. estimators and $G_n^{(m)}(z_n, \theta)$ denotes the m-th derivative relative to θ of the log-likelihood function. In the case $k=1$, $T_{n,1}$ means T_n.

The purpose of this paper is to investigate asymptotic sufficiency of a statistic constructed by m.l. estimators in the following cases. Let $x_1, \cdots, x_n$ be independent and identically distributed random variables with common density $p(x-\theta)$, $-\infty<x, \theta<\infty$, where θ is an unknown translation parameter and $p(x)$ is uniformly continuous and positive only on the interval $(0, \infty)$. We shall consider here two cases.

Case (i): $p(x)\sim\alpha x$ as $x\to+0$, where $\alpha>0$.

Case (ii): $p(x)\sim\alpha x^{1+\beta}$ as $x\to+0$, where $\alpha, \beta>0$.

It is assumed that in Case (i) Fisher's information number is infinite. Let $\hat{\theta}_n$ denote m.l. estimator of θ for the sample size n. In this case, Takeuchi [16] and Woodroofe [20] proved the asymptotic normality of $\sqrt{\frac{1}{2}\alpha n\log n}\,(\hat{\theta}_n-\theta)$ and the speed of convergence to the standard normal distribution was given by Matsuda [4]. Moreover, it was shown by. Takeuchi [16] and Weiss and Wolfowitz [19] that $\hat{\theta}_n$ is an asymptotically efficient estimator of θ.

In Case (ii), it is well known that if Fisher's information number J is finite, then the distribution of $\sqrt{Jn}\,(\hat{\theta}_n-\theta)$ converges weakly to the standard normal distribution. The order of convergence to normality is $o(n^{-\nu/2})$ for every $\nu<\beta$ if $\beta\leqq 1$ and $O(n^{-1/2})$ if $\beta>1$ (see Matsuda [3] and cf. also Pfanzagl [11]).

In both cases, Mita [9] showed that m.l. estimator is asymptotically sufficient up to order $o(1)$. For $n, k\in N$ define $\hat{\theta}_{n,k}=(\hat{\theta}_n, G_n^{(2)}(z_n, \hat{\theta}_n), \cdots, G_n^{(k)}(z_n, \hat{\theta}_n))$, where $\hat{\theta}_{n,1}$ means $\hat{\theta}_n$. We shall show that in Case (i) the statistic $\hat{\theta}_{n,k}$ is asymptotically sufficient up to order $o((\log n)^{-\nu})$ for every $\nu<(k+1)/(k+3)$ and that in Case (ii) $\hat{\theta}_{n,k}$ is asymptotically sufficient up to order $o(n^{-\nu})$ for every