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ASYMPTOTIC SUFFICIENCY I: REGULAR CASES

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1. Introduction. The concept of asymptotic sufficiency of maximum likelihood (m.l.) estimator is due to Wald [16] and this work was succeeded by LeCam [4] and Pfanzagl [10]. Higher order asymptotic sufficiency has been subsequently studied by Ghosh and Subramanyam [3], Michel [7] and Suzuki [14], [15].

Let Θ be an open subset of the s-dimensional Euclidean space. Suppose that $x_1, \cdots, x_n$ are independent and identically distributed random variables with joint distribution $P_{n,\theta}$, $\theta \in \Theta$, which has a constant support and satisfies certain regularity conditions. For $\theta \in \Theta$ and $z_n = (x_1, \cdots, x_n)$ let $G_n^{(m)}(z_n, \theta)$ denote the m-th derivative relative to θ of the log-likelihood function. In Michel [7], it was shown that for $k \geqq 3$ a statistic $T_{n,k} = (T_n, G_n^{(2)}(z_n, T_n), \cdots, G_n^{(k)}(z_n, T_n))$, where $\{T_n\}$ is a sequence of asymptotic m.l. estimators of order $o(n^{-(k-2)/2})$ (see Definition in Section 3), is asymptotically sufficient up to order $o(n^{-(k-2)/2})$ in the following sense: For each $n \in N$, $T_{n,k}$ is sufficient for a family $\{Q_{n,\theta}; \theta \in \Theta\}$ of probability distributions and for every compact subset K of Θ

$$\sup_{\theta \in K} ||P_{n,\theta} - Q_{n,\theta}|| = o(n^{-(k-2)/2}),$$

where $||\cdot||$ means the total variation of a measure. Suzuki [14], [15] also showed that for $k \in N$ a statistic $(\hat{\theta}_n, G_n^{(1)}(z_n, \hat{\theta}_n), \cdots, G_n^{(k)}(z_n, \hat{\theta}_n))$, where $\hat{\theta}_n$ is a reasonable estimator including m.l. estimator, is asymptotically sufficient up to order $o(n^{-(k-1)/2})$ under a stronger moment condition than in Michel [7].

In this paper we give a refinement of their results on higher order asymptotic sufficiency. Our result includes that (1) $T_{n,k} = (T_n, G_n^{(2)}(z_n, T_n), \cdots, G_n^{(k)}(z_n, T_n))$ is asymptotically sufficient up to order $O(n^{-k/2})$ for any sequence $\{T_n\}$ of asymptotic m.l. estimators of order $O(n^{-k/2})$ and (2) a sequence of asymptotic m.l. estimators of order $O(n^{-r/2})$ with some $r \in (0, 1)$ is asymptotically sufficient up to order $O(n^{-r/2})$ under mild moment conditions for the first and the second derivatives of the log-likelihood function.

In the case $k=1$, Pfanzagl ([10], Theorem 1) proved that a sequence of estimators with properties analogous to those of asymptotic m.l. estimators of order $O(n^{-1/2})$ is asymptotically sufficient up to order $O(n^{-1/2})$, and showed in