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Osaka J. Math.
21 (1984), 225-237

QUASICONFORMAL METRIC AND ITS APPLICATION TO QUASIREGULAR MAPPINGS

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(Received September 14, 1982)
(Revised May 21, 1983)

The quasiconformal metric introduced by Kuusalo [5] seems to me useful for studying the n-dimensional quasiregular mappings but has not ever been fully utilized in these connections except what are found in V.M. Gol'dstein-S.K. Vodop'yanov [2] and H. Tanaka [14].

In this paper we shed light on some features of quasiconformal metrics on subdomains of $\bar{R}^n$ and apply those to quasiregular mappings to obtain several important properties of them, among others, a characterization for quasiregularity which comes to a generalization of the result in O. Martio, S. Rickman and J. Väisälä [6, Theorem 7.1]. Most of the statements in the sequel remain to hold in $\bar{R}^n$, but we often confine ourselves to R^n in order to avoid inessential complexities in technique.

1. Notations and terminologies

R^n ($n \geq 2$): the n-dimensional euclidean space.
$\bar{R}^n$: the one point compactification of R^n.
m_α: the α-dimensional Hausdorff measure.
$m = m_n$: the n-dimensional Lebesgue measure.
q: the spherical metric.

For a point $x \in R^n$, the coordinates of x are denoted by $x^1, \cdots, x^n$ and $|x|$ is the euclidean norm.

Let E be a subset of $\bar{R}^n$, then $\bar{E}$, ∂E, E^c denote the closure, the boundary, the complement of E respectively, all taken with respect to $\bar{R}^n$.

Given two sets $E, F \subset R^n$, $d(E, F)$ is the euclidean distance between E and F, $d(E)$ is the euclidean diameter of E and $E \setminus F$ is the set-theoretical difference.

Suppose given a non-empty compact proper subset E of $\bar{R}^n$ and an open set $G \subset \bar{R}^n$, including E, then we call the pair (E, G) a condenser and we may define the (conformal) capacity cap(E, G) as the (conformal) modulus of the family of all paths connecting E and ∂G in G (cf. [3]). If $E = \phi$ or $\partial G = \phi$, then we set cap$(E, G) = 0$.