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MULTI-PRODUCTS OF FOURIER INTEGRAL OPERATORS AND THE FUNDAMENTAL SOLUTION FOR A HYPERBOLIC SYSTEM WITH INVOLUTIVE CHARACTERISTICS

Dedicated to the memory of Professor Hitoshi Kumano-go

KAZUO TANIGUCHI

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Introduction. Let $\mathcal{L}$ be a hyperbolic system with the diagonal principal part

$$(1) \qquad \mathcal{L} = D_t - \begin{bmatrix} \lambda_1(t, X, D_x) & & 0 \\ & \ddots & \\ 0 & & \lambda_l(t, X, D_x) \end{bmatrix} + (b_{mk}(t, X, D_x)) .$$

In order to consider the propagation of singularities of solutions of an equation $\mathcal{L}U(t)=0$, we frequently employ a method of constructing the fundamental solution $\boldsymbol{E}(t, s)$ and investigating its properties. In Kumano-go-Taniguchi-Tozaki [11] and Kumano-go-Taniguchi [10] the fundamental solution $\boldsymbol{E}(t, s)$ of the hyperbolic system $\mathcal{L}$ has been constructed in the form

$$(2) \qquad \begin{aligned} \boldsymbol{E}(t, s) = \boldsymbol{I}_\phi(t, s) + \int_s^t \boldsymbol{I}_\phi(t, \theta)\{\boldsymbol{W}_\phi(\theta, s) \\ + \sum_{\nu=2}^{\infty} \int_s^\theta \int_s^{t_1} \cdots \int_s^{t_{\nu-2}} \boldsymbol{W}_\phi(\theta, t_1) \boldsymbol{W}_\phi(t_1, t_2) \cdots \\ \times \boldsymbol{W}_\phi(t_{\nu-1}, s) dt_{\nu-1} \cdots dt_1\} d\theta \qquad (t_0=\theta) , \end{aligned}$$

where $\boldsymbol{I}_\phi(t, s)$ and $\boldsymbol{W}_\phi(t, s)$ are $l \times l$ matrices of Fourier integral operators $P_\phi(t, s)$ defined by $P_\phi(t, s)u = \int e^{i\phi(t,s;x,\xi)} p(t, s; x, \xi)\hat{u}(\xi) d\xi$. The expression (2) is obtained by constructing, first, an approximate fundamental solution $\boldsymbol{I}_\phi(t, s)$ and next applying the method of the successive approximation. When we want to derive some properties of $\boldsymbol{E}(t, s)$ from (2), it is necessary to estimate the multi-product

$$(3) \qquad \tilde{Q}_{\nu+1} = P_{1,\phi_1} P_{2,\phi_2} \cdots P_{\nu+1,\phi_{\nu+1}}$$