

STEFAN PROBLEMS WITH THE UNILATERAL BOUNDARY CONDITION ON THE FIXED BOUNDARY IV

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Contents

- 0. Introduction
- 1. Statements of main results
- 2. Proof of Theorem 1, 2 and 3
- 3. Comparison theorems
- 4. A priori estimates of the free boundary $s(t)$
- 5. A priori estimates of $u_x(0, t)$ and $u_x(1, t)$
- 6. Weak formulation of the Stefan problem (S)
- 7. Proof of Theorem 4, 5 and 6
- 8. Proof of Corollary 1, 2, 3 and 4
- 9. Some examples
- References

0. Introduction

We consider the behavior of the solution of the following one-dimensional two phase Stefan problem with the unilateral boundary condition on the fixed boundary: Given the initial data, l and $\phi(x)$, find a critical time T^* , and the two functions $s=s(t)$ and $u=u(x, t)$ defined on $[0, T^*]$ such that

$$(S) \left\{ \begin{array}{l} (0.1) \quad s(0) = l, 0 < s(t) < 1 \quad (0 \leq t < T^*), \\ (0.2) \quad u_{xx} - c_0 u_t = 0 \quad (0 < x < s(t), 0 < t < T^*), \\ (0.3) \quad u_{xx} - c_1 u_t = 0 \quad (s(t) < x < 1, 0 < t < T^*), \\ (0.4) \quad \begin{array}{l} (a) \quad u_x(0, t) \in \gamma_0(u(0, t)) \quad (0 < t < T^*), \\ (b) \quad -u_x(1, t) \in \gamma_1(u(1, t)) \quad (0 < t < T^*), \end{array} \\ (0.5) \quad \begin{array}{l} (a) \quad u(x, 0) = \phi(x) \quad (0 < x < l), \\ (b) \quad u(x, 0) = \phi(x) \quad (l < x < 1), \end{array} \\ (0.6) \quad u(s(t), t) = 0 \quad (0 < t \leq T^*), \\ (0.7) \quad b\dot{s}(t) = -u_x^-(s(t), t) + u_x^+(s(t), t) \quad (0 < t < T^*). \end{array} \right.$$