Berman, S.M.
Osaka J. Math.
21 (1984), 133-147

UNBOUNDEDNESS OF SAMPLE FUNCTIONS OF STOCHASTIC PROCESSES WITH ARBITRARY PARAMETER SETS, WITH APPLICATIONS TO LINEAR AND l_p-VALUED PARAMETERS*

SIMEON M. BERMAN

(Received August 3, 1982)

1. Introduction and summary

Let T be a pseudometric space, and let $X(t)$, $t\in T$, be a real valued stochastic process on some probability space. There has been much recent interest in conditions for the continuity or boundedness of the sample functions stated in terms of the finite-dimensional distributions of the process and their relation to the pseudometric. On the other hand, one of my interests has been the search for conditions under which $X(t)$, $t\in T$, is unbounded, or has even more drastically irregular behavior. The main tool in this analysis is the local time of the process. The theme of this work has been that the smoothness of the local time implies the irregularity of the sample function. In the current paper, in particular, we use the result that if the local time is an analytic function of its spatial variable, then the sample function spends positive time in every set of positive measure, so that it is unbounded on T. This was used in [2] to supplement the Beljaev dichotomy theorem for stationary Gaussian processes [1]: In the noncontinuous case, the sample functions are often not only unbounded, but have the property of the Carathéodory function [2].

The current work extends the latter results to more general, not necessarily Gaussian processes. Suppose that there is a nonnegative function $d(s, t)$ on T^2 such that $d(s, t)>0$ for $s\neq t$, and that the density function of the random variable,

$$\frac{X(s)-X(t)}{d(s, t)}, \tag{1.1}$$

is uniformly sufficiently smooth in a precise sense for all $s\neq t$; and suppose that there is a specified nonincreasing function $K(u)$, $u>0$, determined by the

* This paper represents results obtained at the Courant Institute of Mathematical Sciences, New York University, under the sponsorship of the National Science Foundation, Grant MCS 82 01119.