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ON THE MARTINGALE PROBLEM FOR GENERATORS OF STABLE PROCESSES WITH PERTURBATIONS

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0. Introduction

We say that a probability measure P_x on $D(\boldsymbol{R}_+\to\boldsymbol{R}^d)$ solves the martingale problem for a Lévy type generator L starting from x at time $t=0$ if

$$M_t^f = f(X_t)-f(x)-\int_0^t Lf(X_s)ds$$

is a P_x-martingale with $M_0^f=0$ for all test functions f on $\boldsymbol{R}^d$. The martingale problem for second order elliptic differential operators was studied by Stroock and Varadhan [7]. It was Grigelionis [3] who first gave a martingale formulation of Markov processes associated with certain integro-differential operators. Komatsu [4], Tsuchiya [9] and Stroock [8] discussed the existence and the uniqueness of solutions for the martingale problem associated with a Lévy type generator L. The existence was proved in [4] and [8] under a certain continuity condition. The uniqueness was shown in [4] and [8] in a context that the principal part of L is a second order elliptic differential operator. Tsuchiya [9] proved the uniqueness in the case where the principal part of L is the generator of an isotropic stable processes with index α $(1\leqq\alpha<2)$ and the perturbation part of L has the upper index $\beta<\alpha$ (for the precise meaning, see the remark following assumption [A_2] in section 2). The purpose of this paper is to improve the results in [9] by making use of the theory of singular integrals in Calderôn and Zygmund [1].

Let $\exp[t\psi^{(\alpha)}(\xi)]$ be the characteristic function of a stable process with index α. Then $\psi^{(\alpha)}(\xi)$ is a homogeneous function of index α, and the generator $A^{(\alpha)}$ of the stable process is given by

$$A^{(\alpha)}f(x) = \mathcal{F}^{-1}[\psi^{(\alpha)}\mathcal{F}f]\,(x)\,.$$

In case $1<\alpha<2$, the operator $A^{(\alpha)}$ has the following expression

$$A^{(\alpha)}f(x) = \int[f(x+y)-f(x)-y\cdot\partial f(x)]M^{(\alpha)}(dy)\,,$$

where $M^{(\alpha)}(dy)$ is a measure on $\boldsymbol{R}^d\setminus\{0\}$ such that there is a finite measure