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ON THE ROBERTELLO INVARIANTS OF PROPER LINKS

Dedicated to Professor Minoru Nakaoka on his 60th birthday

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Robertello's invariant of a classical knot in [9] was generalized by Gordon in [2] to an invariant of a knot in a Z-homology 3-sphere, and by the author in [5] to an invariant, $\delta(k\subset S)$, of a knot k in a Z_2-homology 3-sphere S. In this paper, we shall generalize this invariant to two mutually related invariants, $\delta_0(L\subset S)$ and $\delta(L\subset S)$, of a proper link L in a Z_2-homology 3-sphere S. In the case of a classical proper link, this δ_0-invariant can be considered as an invariant suggested by Robertello in [9, Theorem 2]. A difference between $\delta_0(L\subset S)$ and $\delta(L\subset S)$ is that $\delta_0(L\subset S)$ is generally an oriented link type invariant, but $\delta(L\subset S)$ is an unoriented link type invariant. A proper link in a Z_2-homology 3-sphere (which is not a Z-homology 3-sphere) naturally occurs when considering a branched cyclic covering of a 3-sphere, branched along a certain proper link. (If the number of the components of the link is ≥ 2, the branched covering space can not be a Z-homology 3-sphere by the Smith theory.) So, we consider a proper link $\tilde{L}$ in a Z_2-homology 3-sphere $\tilde{S}$, obtained from a proper link L in a Z_2-homology 3-sphere S by taking a branched cyclic covering, branched along L. When the covering degree is prime, we shall establish a relationship between $\delta(\tilde{L}\subset\tilde{S})$ and $\delta(L\subset S)$ and then a relationship between $\delta_0(\tilde{L}\subset\tilde{S})$ and $\delta_0(L\subset S)$.

In Section 1 we define and discuss the slope of a link in a 3-manifold as a generalization of the slope of a knot in a 3-manifold, introduced in [5]. In Section 2 the δ_0-invariant and the δ-invariant are defined and studied. Section 3 deals with relationships between $\delta(\tilde{L}\subset\tilde{S})$ and $\delta(L\subset S)$ and between $\delta_0(\tilde{L}\subset\tilde{S})$ and $\delta_0(L\subset S)$.

Throughout this paper spaces and maps will be considered in the piecewise linear category, and notations and conventions will be the same as those of [5] unless otherwise stated.

1. The slope of a link in a 3-manifold

Let M be a connected oriented 3-manifold. Let L be an oriented link with r components in the interior of M. Let $o(L)$ denote the order (≥ 1) of