Morimoto, M. and Iizuka, K.
Osaka J. Math.
21 (1984), 59–69

EXTENDIBILITY OF G-MAPS TO PSEUDO-EQUIVALENCES TO FINITE G-CW-COMPLEXES WHOSE FUNDAMENTAL GROUPS ARE FINITE

Dedicated to Professor Minoru Nakaoka on his sixtieth birthday

MASAHARU MORIMOTO AND KUNIHIKO IIZUKA

(Received August 4, 1982)

0. Introduction

In this paper we let G be a finite group. A. Assadi [2] and R. Oliver-T. Petrie [6] treated the following question. What is a necessary and sufficient condition, for given finite G-CW-complexes X and Y and a G-map $f: X \rightarrow Y$, to extend f to a quasi-equivalence $f': X' \rightarrow Y$ (with some reservations)? Here a G-map is called a quasi-equivalence if it induces isomorphisms of fundamental groups and of integral homology groups. We apply the Oliver-Petrie theory to covering spaces to give a necessary and sufficient condition so that we may extend above f to a pseudo-equivalence $f'': X'' \rightarrow Y$ (with some reservations), when $\pi_1(Y)$ is finite.

We take Oliver-Petrie [6] as our general reference and use their terms and notations.

Let Y be a finite connected G-complex. Then $\tilde{G}=\pi_1(EG\times_G Y)$ acts on the universal covering space $\tilde{Y}$ of Y as is shown in section 1 (compare the action with that of D. Anderson [1]). Assume $\pi_1(Y)$ is finite. Then $\tilde{G}$ is finite, so we have a $\tilde{G}$-poset $\tilde{\Pi}=\Pi(\tilde{Y})$ and a G-poset $\Pi=\Pi(Y)$. In section 3 we give a one to one correspondence T from the set of G-families in Π to the set of $\tilde{G}$-families in $\tilde{\Pi}$, and an isomorphism ν from $\Omega(\tilde{G}, \tilde{\Pi})$ to $\Omega(G, \Pi)$. A subgroup $\Delta_h(G, Y, \mathcal{F})$ of $\Delta(G, \mathcal{F})$ is defined by $\Delta_h(G, Y, \mathcal{F})=\nu(\Delta(\tilde{G}, T(\mathcal{F})))$. Under certain conditions $\Delta_h(G, Y, \mathcal{F})$ agrees with the set

$$\{[M_f]\in\Omega(G, \Pi)\,|\,f: X\rightarrow Y \text{ is a pseudo-equivalence such that } X^+ \text{ is an } \mathcal{F}\text{-complex}\}$$

(see Proposition 4.1), where M_f is the mapping cone of f.

Our main results are:

Theorem 1. *Let X be a finite G-complex, Y a finite connected G-complex with finite $\pi_1(Y)$, $f: X \rightarrow Y$ a skeletal G-map, and $\mathcal{F}\subset\Pi$ any connected G-*