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CLASSIFICATION OF INVARIANT COMPLEX STRUCTURES ON IRREDUCIBLE COMPACT SIMPLY CONNECTED COSET SPACES

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Introduction

A compact simply connected homogeneous Kähler manifold is represented as a Kähler coset space G/U, where G is a compact connected semisimple Lie group and U is the centralizer of a toral subgroup S in G. Conversely, let G be a compact connected semisimple Lie group and U the centralizer of a toral subgroup in G. Then, G/U is a compact simply connected C^∞-manifold and carries a G-invariant complex structure. Moreover any G-invariant complex structure on G/U admits a G-invariant Kähler metric. In this paper, we shall consider the problem of classifying, up to equivalence, all G-invariant complex structures on the coset space G/U. Borel-Hirzebruch [2] showed that G-invariant complex structures on G/U are unique up to equivalence if U is a maximal torus of G or if U is a subgroup with one-dimensional center.

We shall consider exclusively the case where G is a simple compact Lie group and in this case we say that the coset space G/U is irreducible. We shall classify all G-invariant complex structures on an irreducible compact simply connected coset space G/U up to equivalence. An equivalence class of G-invariant complex structures on G/U gives rise to a pair of a simple root systems (π, π_0) such that π_0 is a subsystem of π and this pair is determined uniquely up to equivalence. Here two pairs (π, π_0) and (π', π_0') are said to be equivalent if there is an isomorphism between the systems π and π' which maps π_0 to π_0'. Our classification will then be reduced to that of classifying, up to equivalence, all pairs (π, π_0) associated to G/U and in this way we shall count up the number of equivalence classes of G-invariant complex structures on G/U.

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1. *G*-invariant complex structures

Let G be a Lie group and U a closed subgroup of G. We denote by $\mathfrak{g}$