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ON THE STRICT CLASS NUMBER OF $Q(\sqrt{2p})$ MODULO 16, $p\equiv 1$ (mod 8) PRIME

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Let $p\equiv 1 \pmod 8$ be prime so that there are integers a, b, c, d, e, f with

$$(1)\qquad \begin{cases} p = a^2+b^2 = c^2+2d^2 = e^2-2f^2 \\ a\equiv 1 \pmod 4,\ b\equiv 0 \pmod 4,\ c\equiv 1 \pmod 4,\ d\equiv 0 \pmod 2, \\ \qquad e\equiv 1 \pmod 4,\ f\equiv 0 \pmod 4. \end{cases}$$

Throughout this note we consider only those primes p for which the strict class number $h^+(8p)$ of the real quadratic field $\boldsymbol{Q}(\sqrt{2p})$ (of discrimanant $8p$) satisfies

$$(2)\qquad h^+(8p)\equiv 0 \pmod 8.$$

These primes have been characterized by Kaplan [4]. Indeed such primes must satisfy [5]

$$(3)\qquad \begin{cases} p\equiv 1 \pmod{16},\ a\equiv 1 \pmod 8,\ b\equiv 0 \pmod 8,\ c\equiv 1 \pmod 8,\ \left(\frac{c}{p}\right) = 1, \\ d\equiv 0 \pmod 4,\ e\equiv 1 \pmod 8,\ \left(\frac{e}{p}\right) = +1. \end{cases}$$

In this note we give a new determination of $h^+(8p)$ modulo 16, and compare it with the determination given by Yamamoto in [15].

We begin by introducing some notation. We denote the fundamental unit (>1) of $\boldsymbol{Q}(\sqrt{2p})$ by η_{2p}. As one and only one of the equations $V^2-2pW^2 =-1$, -2, or $+2$ is solvable in integers V, W, we define

$$E_p = \begin{cases} -1, & \text{if } V^2-2pW^2 = -1 \text{ solvable}, \\ -2, & \text{if } V^2-2pW^2 = -2 \text{ solvable}, \\ +2, & \text{if } V^2-2pW^2 = +2 \text{ solvable}. \end{cases}$$

Clearly the norm $N(\eta_{2p})$ of η_{2p} satisfies

$$N(\eta_{2p}) = \begin{cases} +1, & \text{if } E_p = \pm 2, \\ -1, & \text{if } E_p = -1. \end{cases}$$