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Osaka J. Math.
21 (1984), 1-22

DIVISIBILITY BY 16 OF CLASS NUMBER OF QUADRATIC FIELDS WHOSE 2-CLASS GROUPS ARE CYCLIC

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(Received August 5, 1982)

0. Introduction. Let $K=\boldsymbol{Q}(\sqrt{D})$ be the quadratic field with discriminant D, and $H(D)$ and $h(D)$ be the ideal class group of K and its class number respectively. The ideal class group of K *in the narrow sense* and its class number are denoted by $H^+(D)$ and $h^+(D)$ respectively. We have $h^+(D)=2h(D)$, if $D>0$ and the fundamental unit ε_D (>1) has the norm 1, and $h^+(D)=h(D)$, otherwise. We assume, throughout the paper, that $|D|$ has just two distinct prime divisors, written p and q, so that the 2-class group of K (i.e. the Sylow 2-subgroup of $H^+(D)$ because we mean in the narrow sense) is cyclic. Then the discriminant D can be written uniquely as a product of two prime discriminants d_1 and d_2, $D=d_1d_2$, such that $p|d_1$ and $q|d_2$ (cf. [16], for example).

By Redei and Reichardt [13] (cf. proposition 1.2 below), $h^+(D)$ is divisible by 4 if and only if D belongs to one of the following 6 types:

(R1) $D=pq$, $d_1=p$, $d_2=q$, $p\equiv q\equiv 1 \pmod 4$, *and* $\left(\frac{p}{q}\right)=1$ $\left(=\left(\frac{q}{p}\right)\right.$ *by reciprocity*);

(R2) $D=8q$, $d_1=8$ $(p=2)$, $d_2=q$, *and* $q\equiv 1 \pmod 8$;

(I1) $D=-pq$, $d_1=-p$, $d_2=q$, $p\equiv 3 \pmod 4$, $q\equiv 1 \pmod 4$, *and* $\left(\frac{-p}{q}\right)=1$ $\left(=\left(\frac{q}{p}\right)\right.$ *by reciprocity*);

(I2) $D=-8p$, $d_1=-p$, $d_2=8$ $(q=2)$, *and* $p\equiv 7 \pmod 8$;

(I3) $D=-8q$, $d_1=-8$ $(p=2)$, $d_2=q$, *and* $q\equiv 1 \pmod 8$;

(I4) $D=-4q$, $d_1=-4$ $(p=2)$, $d_2=q$, *and* $q\equiv 1 \pmod 8$;

where $\left(\frac{}{}\right)$ is the Legendre-Jacobi-Kronecker symbol.

Conditions for $h^+(D)$ to be divisible by 8 have been given by several authors for each case or cases ([1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 15]). Some of them are reformulated in section 3. The purpose of this paper is to give some conditions for the divisibility by 16 of $h^+(D)$ for each case (cf. theorems 5.4, 5.5, 5.6, 5.7, 5.8, and 6.7). The main ideas were announced in [18] and [19].

While in preparation of the manuscript P. Kaplan informed me that theorem 6.7 was proved also by K.S. Williams with a different method and furthermore he gave a congruence for $h(-4q)$ modulo 16 ([17]).

* Reseach supported partly by Grant-in-Aid for Scientific Research.