The mathematical work of Y. Matsushima and its development

Shoshichi KOBAYASHI*)

In the past thirty years, differential geometry has undergone an enormous change with infusion of topology, Lie theory, complex analysis, algebraic geometry and partial differential equations. Professor Matsushima played a leading role in this transformation by bringing new techniques of Lie groups and Lie algebras into the study of real and complex manifolds. He was a man of great insight; the problems he has worked on and the results he has obtained continue to be sources of inspiration for us geometers. When I was asked by the editorial board of the journal to write about Matsushima's work, I decided therefore to contribute something that would serve as a guide for students of geometry who wish to work in the area explored by him. I only hope that I am not leading them astray by committing too many errors.

In relating other papers to Matsushima's work, I tried, whenever possible, to cite for reference books and survey articles from which unquoted original papers can be easily traced back. I am grateful to Professors S. Murakami and K. Nomizu for calling my attention to several papers I overlooked in my first draft.

I. Lie theory

(1) Lie algebras and Lie groups, [a, 1, 3, 4, 5, 6, 7, 8, 9, 10].

Upon his graduation from the Osaka Imperial University in the fall of 1942, Matsushima became an Assistant in the Mathematical Institute of the newly established Nagoya Imperial University. Beginning with his first paper [1] in which he disproved a conjecture of Zassenhaus (that every semisimple Lie algebra L of prime characteristic with $L=[L,L]$ is a direct sum of simple ideals), he spent the next ten years working on Lie groups and Lie algebras. During and also for several years after the Second World War, it was extremely difficult for Japanese mathematicians to gain access to foreign periodicals or to have their papers published. In [4], unaware of Chevalley's paper [1] Matsushima gave a separate algebraic proof on conjugacy of Cartan sublagebras. In [3] written with the knowledge of Chevalley's paper [2] only through the Mathematiacl Reviews, Matsushima gave new proofs to Chevalley's main results on replicas of matrices. (One can surmise adverse conditions at that time from the fact that the Proceedings of Japan Academy, vol. 23 (1947), containing