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ON THE SPACE OF MINIMAL SURFACES WITH BOUNDARIES

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0. Introduction

In this paper we are concerned with the space of minimal surfaces with boundaries.

For each Jordan curve Γ in R^n the existence of minimal surfaces spanned by Γ is well known. We are interested in the behavior of such minimal surfaces as their boundary contours vary. In what case a minimal surface moves smoothly according as its boundary varies? And in what case a minimal surface vanishes or branches out into two or more minimal surfaces as a result of a perturbation of its boundary? Although we have not yet succeeded to answer such questions perfectly, we can prove a certain local property of the space of minimal surfaces, which is the first step of our attempt to solve the above mentioned problem. In this paper we shall derive it.

Let us restrict ourselves to minimal surfaces spanned by $C^{r,\alpha}$-curves ($r\geq 2$, $0<\alpha<1$). Consider a regular minimal surface f_0 spanned by a regular analytic Jordan curve. Then we shall show that in a neighborhood of f_0 the set of all minimal surfaces is isomorphic in a very natural way to an open set of an infinite dimensional Banach space (Corollary 4.6).

There exist some interesting results of other authors relating to our concerns. Using Sobolev spaces instead of $C^{r,\alpha}$, Tromba [9] proved the existence of an open and dense subset γ of all "fine embeddings" in $H^k(S^1, R^n)$ ($k\geq 7$), such that the set of minimal surfaces spanned by $\Gamma\in\gamma$ are "differentiable functions" of Γ. On the other hand Böhme [2] derived a more general result, but only in case $n=3$, employing the classical Weierstrass representation formula for minimal surfaces. More recently Böhme and Tromba [3] proved the existence of an open and dense subset $\hat{A}$ of all H^r embeddings with total curvature bounded by $\pi(s-2)$ (for integers r and s, $r\gg s\geq 7$) such that the number of all minimal surfaces spanned by $\Gamma\in\hat{A}$ is finite and these minimal surfaces are "stable" under perturbations of Γ.

If the boundary of our surface f_0 is contained in the Böhme's or Tromba's set of curves, our result may be almost trivial. But it should be noted that